Modern Physics Letters B
Vol. 28, No. 32 (2014) 1450252 (12 pages)

DOI: 10.1142/S0217984914502522

Resistance calculation of the decorated centered cubic networks: Applications of the Green's function

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Received 25 September 2014
Accepted 13 November 2014
Published 31 December 2014

The effective resistance between any pair of vertices (sites) on the three-dimensional decorated centered cubic lattices is determined by using lattice Green's function method. Numerical results are presented for infinite decorated centered cubic networks. A mapping between the resistance of the edge-centered cubic lattice and that of the simple cubic lattice is shown

Keywords: Green's function; effective resistance; decorated centered cubic lattices.

1. Introduction

The problem of calculating the two-vertex resistance on an infinite resistor lattice has been investigated before. It is a basic problem in electrical circuit theory. Various approaches[1–6] have been used for tackling this problem. The most elegant and efficient method to study this problem is based on the lattice Green's function, which has been introduced by Cserti[6] and developed by Cserti *et al.*[7] for any infinite lattice structure that is a uniform tiling of d-dimensional space with resistors. Also, the Green's function technique is used to study the perturbed lattices.[18] Based on this approach, several interesting applications were presented.[9–18]

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The problem of computing the two-node resistance on finite resistor networks was considered by Wu.[19] He obtained a general expression for the resistance of a finite resistor graph in terms of the eigenvalues and the eigenvectors of the Kirchhoff matrix. Based on Wu's method several studies have been carried out.[20–22]

In this paper, the Green's function approach[7] is used to study the effective resistance between any two vertices in the following infinite, decorated centered cubic electrical networks: base-centered cubic, side-centered cubic and edge-centered cubic lattice, that have not been considered in the literature.

The rest of the paper is constructed as follows. In Sec. 2, a brief review of the formulation of the methodology of the two-point resistance in infinite uniform tiling introduced in Ref. 7 is given. In Sec. 3, this formulation is applied to the decorated centered cubic networks. Also, some numerical results regarding the resistance between certain pairs of vertices are presented and discussed. A brief conclusion is given in Sec. 4. The results for the resistance in the edge-centered decorated cubic network in terms of the resistances in a simple cubic lattice are appended in Appendix A.

2. Formulation of Two-Vertex Resistance on a Periodic Lattice

In this section, we briefly review the formulation of two-vertex resistance on a periodic lattice of equal resistors (for more details, see Ref. 7).

Consider a resistor lattice network structure which is a periodic lattice of d-dimensional space with $N_1, N_2, \ldots, N_d$ unit cells along each unit cell vectors $\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_d$. If the unit cell containing s sites (vertices) numbered by $\alpha = 1, 2, \ldots, s$ then there are $sN_1N_2, \ldots, N_d$ vertices in the lattice. Let $\{\mathbf{r}; \alpha\}$ denote any vertex, thus the unit cell and the lattice site can be specified by $\mathbf{r} = (\ell_1\mathbf{a}_1, \ell_2\mathbf{a}_2, \ldots, \ell_d\mathbf{a}_d)$ and α respectively, where $\ell_1, \ell_2, \ldots, \ell_d$ are any integers. Assume, without loss of generality, each resistor of resistance R.

Let the electric potential and current at vertex $\{\mathbf{r}; \alpha\}$ are $\varphi_\alpha(\mathbf{r})$ and $I_\alpha(\mathbf{r})$, respectively. Applying Kirchhoff's current and Ohm's laws at site $\{\mathbf{r}; \alpha\}$, the currents $I_\alpha(\mathbf{r})$ in the unit cell can be written in the form:

$$\sum_{\mathbf{r}',\beta} L_{\alpha\beta}(\mathbf{r}, \mathbf{r}')\varphi_\beta(\mathbf{r}') = -RI_\alpha(\mathbf{r})\,, \tag{1}$$

where $L_{\alpha\beta}(\mathbf{r}, \mathbf{r}')$ is the Laplacian matrix of the network. The potential and current at vertex $\{\mathbf{r}; \alpha\}$ can be written in terms of their Fourier transforms:

$$\varphi_\alpha(\mathbf{r}) = \Omega_0 \int_{-\pi/a_1}^{\pi/a_1} \frac{dk_1}{2\pi} \cdots \int_{-\pi/a_d}^{\pi/a_d} \frac{dk_d}{2\pi} \varphi_\alpha(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{r}}\,, \tag{2}$$

and

$$I_\alpha(\mathbf{r}) = \Omega_0 \int_{-\pi/a_1}^{\pi/a_1} \frac{dk_1}{2\pi} \cdots \int_{-\pi/a_d}^{\pi/a_d} \frac{dk_d}{2\pi} I_\alpha(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{r}}\,, \tag{3}$$

where Ω_0 is volume of unit cell and $\mathbf{k}$ is the wave vector in the reciprocal lattice or Fourier space of the lattice.

Using Eqs. (2) and (3), Eq. (1) may be written as

$$\mathbf{L}(\mathbf{k})\boldsymbol{\varphi}(\mathbf{k}) = -R\mathbf{I}(\mathbf{k})\,, \tag{4}$$

where $\mathbf{L}(\mathbf{k})$ is Fourier transform of the Laplacian matrix ($s \times s$ matrix) and $\boldsymbol{\varphi}(\mathbf{k})$, $\mathbf{I}(\mathbf{k})$ are Fourier transforms (column matrices) of the potential and current. Fourier transform of the Green's function is given by

$$\mathbf{L}(\mathbf{k})\mathbf{G}(\mathbf{k}) = -\mathbf{I}\,, \tag{5}$$

where $\mathbf{I}$ is $s \times s$ identity matrix.

From Eqs. (4) and (5) we have

$$\boldsymbol{\varphi}(\mathbf{k}) = R\mathbf{G}(\mathbf{k})\mathbf{I}(\mathbf{k})\,. \tag{6}$$

Now the resistance between the vertices $\{\mathbf{r}_1;\alpha\}$ and $\{\mathbf{r}_2;\beta\}$ is given by

$$R_{\alpha\beta}(\mathbf{r}_1,\mathbf{r}_2) = \frac{\varphi_\alpha(\mathbf{r}_1) - \varphi_\beta(\mathbf{r}_2)}{I}\,. \tag{7}$$

The computation of the two-point resistance is now reduced to solving (1) or (6) for $\varphi_\alpha(\mathbf{r})$ with the current distribution at any vertex is given by

$$I_\nu(\mathbf{r}) = I\left(\delta_{\alpha,\nu}\delta_{\mathbf{r},\mathbf{r}_1} - \delta_{\beta,\nu}\delta_{\mathbf{r},\mathbf{r}_2}\right)\,. \tag{8}$$

Therefore, using Eqs. (2), (6) and (8) the resistance between the vertices $\{\mathbf{r}_1;\alpha\}$ and $\{\mathbf{r}_2;\beta\}$ defined in Eq. (7) can be written as

$$\begin{aligned} R_{\alpha\beta}(\mathbf{r}_1,\mathbf{r}_2) = \Omega_0 R \int_{-\pi/a_1}^{\pi/a_1}\frac{dk_1}{2\pi}\cdots\int_{-\pi/a_d}^{\pi/a_d}\frac{dk_d}{2\pi} \\ \times\left(G_{\alpha\alpha}(\mathbf{k}) + G_{\beta\beta}(\mathbf{k}) - G_{\alpha\beta}(\mathbf{k})e^{-i\mathbf{k}(\mathbf{r}_2-\mathbf{r}_1)} - G_{\alpha\beta}(\mathbf{k})e^{i\mathbf{k}(\mathbf{r}_2-\mathbf{r}_1)}\right). \end{aligned} \tag{9}$$

It has been pointed out in Ref. 7 that the lattice structure can be deformed into d-dimensional hypercubic lattice without changing the two-point resistance of a resistor network. For hypercubic lattice the unit cell vectors are orthogonal and have the same magnitude. Thus, writing $\mathbf{r}_2 - \mathbf{r}_1 = \sum_{i=1}^{d}\ell_i\mathbf{a_i}$ and $\mathbf{k}\cdot\mathbf{a_i} = \theta_i$ with $\Omega_0 = a^d$ then, Eq. (9) becomes

$$\begin{aligned} R_{\alpha\beta}(\ell_1,\ldots,\ell_d) = R\int_{-\pi}^{\pi}\frac{d\theta_1}{2\pi}\cdots\int_{-\pi}^{\pi}\frac{d\theta_d}{2\pi}\big\{G_{\alpha\alpha}(\theta_1,\ldots,\theta_d) + G_{\beta\beta}(\theta_1,\ldots,\theta_d) \\ - G_{\alpha\beta}(\theta_1,\ldots,\theta_d)e^{-i(\ell_1\theta_1+\ell_2\theta_2+\cdots\theta_d\ell_d)} \\ - G_{\alpha\beta}(\theta_1,\ldots,\theta_d)e^{i(\ell_1\theta_1+\ell_2\theta_2+\cdots\theta_d\ell_d)}\big\}\,. \end{aligned} \tag{10}$$

For three-dimensional hypercubic lattices as in our cases the resistance between the origin $\{\mathbf{0};\alpha\}$ and vertex $\{\ell_1,\ell_2,\ell_3;\beta\}$ is given by

$$R_{\alpha\beta}(\ell_1,\ell_2,\ell_3) = R\int_{-\pi}^{\pi}\frac{d\theta_1}{2\pi}\int_{-\pi}^{\pi}\frac{d\theta_2}{2\pi}\int_{-\pi}^{\pi}\frac{d\theta_3}{2\pi}\{G_{\alpha\alpha}(\theta_1,\theta_2,\theta_3)+G_{\beta\beta}(\theta_1,\theta_2,\theta_3) - G_{\alpha\beta}(\theta_1,\theta_2,\theta_3)e^{-i(\ell_1\theta_1+\ell_2\theta_2+\theta_3\ell_3)} - G_{\alpha\beta}(\theta_1,\theta_2,\theta_3)e^{i(\ell_1\theta_1+\ell_2\theta_2+\theta_3\ell_3)}\}. \tag{11}$$

3. Applications: Resistances on Decorated Centered Cubic Networks

In this section, we calculate the resistance between two arbitrary vertices in infinite decorated centered cubic electrical networks with equal resistances R.

3.1. *Base-centered cubic network*

The so-called base-centered cubic lattice is a three-dimensional lattice. Besides the resistors on the edges of the cube there are resistors between the centers of its two bases (horizontal faces) to the corners of these bases as shown in Fig. 1. The unit cell is a cube containing two vertices labeled by $\alpha = A$ and B: A at origin (one of the corners) of the cube, and B at the center of one horizontal face of the cube nearest the corner vertex. The orthogonal unit cell vectors are $\mathbf{a}_1, \mathbf{a}_2$ and $\mathbf{a}_3$ along the edges of the cube.

Applying Kirchhoff's current rule to vertices $\{\mathbf{r};A\}$ and $\{\mathbf{r};B\}$ and then, using Ohm's law the currents at these vertices, respectively are given by

$$I_A(\mathbf{r}) = \frac{\varphi_A(\mathbf{r})-\varphi_A(\mathbf{r}+\mathbf{a}_1)}{R}+\frac{\varphi_A(\mathbf{r})-\varphi_A(\mathbf{r}-\mathbf{a}_1)}{R}+\frac{\varphi_A(\mathbf{r})-\varphi_A(\mathbf{r}+\mathbf{a}_2)}{R} + \frac{\varphi_A(\mathbf{r})-\varphi_A(\mathbf{r}-\mathbf{a}_2)}{R}+\frac{\varphi_A(\mathbf{r})-\varphi_A(\mathbf{r}+\mathbf{a}_3)}{R}+\frac{\varphi_A(\mathbf{r})-\varphi_A(\mathbf{r}-\mathbf{a}_3)}{R} + \frac{\varphi_A(\mathbf{r})-\varphi_B(\mathbf{r})}{R}+\frac{\varphi_A(\mathbf{r})-\varphi_B(\mathbf{r}-\mathbf{a}_1)}{R}+\frac{\varphi_A(\mathbf{r})-\varphi_B(\mathbf{r}-\mathbf{a}_2)}{R} + \frac{\varphi_A(\mathbf{r})-\varphi_B(\mathbf{r}-\mathbf{a}_1-\mathbf{a}_2)}{R}, \tag{12}$$

$$I_B(\mathbf{r}) = \frac{\varphi_B(\mathbf{r})-\varphi_A(\mathbf{r})}{R}+\frac{\varphi_B(\mathbf{r})-\varphi_A(\mathbf{r}+\mathbf{a}_1)}{R}+\frac{\varphi_B(\mathbf{r})-\varphi_A(\mathbf{r}+\mathbf{a}_2)}{R} + \frac{\varphi_B(\mathbf{r})-\varphi_A(\mathbf{r}+\mathbf{a}_1+\mathbf{a}_2)}{R}. \tag{13}$$

Substituting the Fourier transforms of the electric potentials and currents given in Eqs. (2) and (3) into (12) and (13) then, changing $\mathbf{k}\cdot\mathbf{a}_i$ to $\theta_i (i = 1,2,3)$ the Fourier transform of the Laplacian matrix $\mathbf{L}(\theta_1,\theta_2,\theta_3)$ is given by

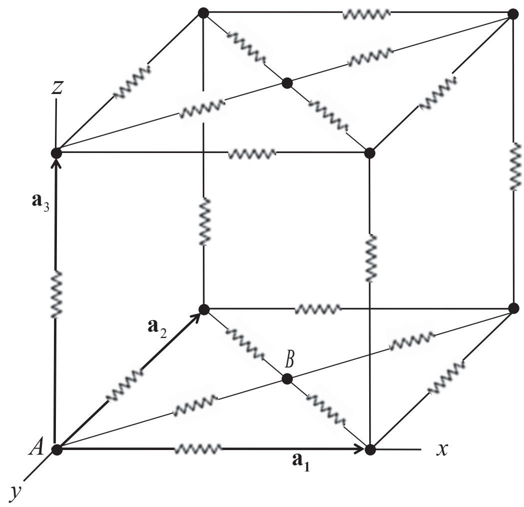

Fig. 1. The resistor network of the base-centered cubic lattice.

$$\mathbf{L}(\theta_1,\theta_2,\theta_3)=\begin{pmatrix}-10+2\cos\theta_1+2\cos\theta_2+2\cos\theta_3 & (1+e^{-i\theta_1})(1+e^{-i\theta_2})\\ (1+e^{i\theta_1})(1+e^{i\theta_2}) & -4\end{pmatrix}. \tag{14}$$

The Green's function matrix $\mathbf{G}(\theta_1,\theta_2,\theta_3)$ corresponding to Fourier transform of Laplacian matrix $\mathbf{L}(\theta_1,\theta_2,\theta_3)$ can be calculated by inverting $\mathbf{L}(\theta_1,\theta_2,\theta_3)$, we get

$$\mathbf{G}(\theta_1,\theta_2,\theta_3)=\frac{1}{\det(\mathbf{L}(\theta_1,\theta_2,\theta_3))}\times\begin{pmatrix}4 & (1+e^{-i\theta_1})(1+e^{-i\theta_2})\\ (1+e^{i\theta_1})(1+e^{i\theta_2}) & 10-2\cos\theta_1-2\cos\theta_2-2\cos\theta_3\end{pmatrix}, \tag{15}$$

where

$$\det(\mathbf{L}(\theta_1,\theta_2,\theta_3))=36-12\cos\theta_1-12\cos\theta_2-8\cos\theta_3-4\cos\theta_1\cos\theta_2 \tag{16}$$

is the determinant of $\mathbf{L}(\theta_1,\theta_2,\theta_3)$.

Using the expression (12), we calculate the resistance between the vertices $\{\mathbf{0};\alpha\}$ and $\{\ell_1,\ell_2,\ell_3;\beta\}$ numerically. As an example, the resistance between adjacent vertices $\{\mathbf{0};A\}$ and $\{\mathbf{0};B\}$ is

$$R_{AB}(0,0,0)=R\int_{-\pi}^{\pi}\frac{d\theta_1}{2\pi}\int_{-\pi}^{\pi}\frac{d\theta_2}{2\pi}\int_{-\pi}^{\pi}\frac{d\theta_3}{2\pi}\times\frac{(12-4\cos\theta_1-4\cos\theta_2-2\cos\theta_1\cos\theta_2-2\cos\theta_3)}{36-12\cos\theta_1-12\cos\theta_2-4\cos\theta_1\cos\theta_2-8\cos\theta_3}. \tag{17}$$

Table 1. Numerical values of the effective resistance $R_{\alpha\beta}(\ell_1, \ell_2, \ell_3)$ in units of R between the vertices $\{\mathbf{0}; \alpha\}$ and $\{\ell_1, \ell_2, \ell_3; \beta\}$ in the base-centered cubic lattice.

$R_{\alpha\beta}(\ell_1, \ell_2, \ell_3)$	The values of $R_{\alpha\beta}(\ell_1, \ell_2, \ell_3)$ in units of R	$R_{\alpha\beta}(\ell_1, \ell_2, \ell_3)$	The values of $R_{\alpha\beta}(\ell_1, \ell_2, \ell_3)$ in units of R
$R_{AA}(1,0,0)$	0.212747	$R_{AB}(0,0,0)$	0.333993
$R_{AA}(0,1,0)$	0.212747	$R_{AB}(1,0,0)$	0.416689
$R_{AA}(0,0,1)$	0.238534	$R_{AB}(0,1,0)$	0.416689
$R_{AA}(1,1,0)$	0.246449	$R_{AB}(0,0,1)$	0.426393
$R_{AA}(1,0,1)$	0.264575	$R_{AB}(1,1,0)$	0.437046
$R_{AA}(0,1,1)$	0.264575	$R_{AB}(1,0,1)$	0.442804
$R_{AA}(1,1,1)$	0.273858	$R_{AB}(0,1,1)$	0.442804
$R_{AA}(2,0,0)$	0.268087	$R_{AB}(1,1,1)$	0.450644
$R_{BB}(1,0,0)$	0.541348	$R_{AB}(2,0,0)$	0.446861
$R_{BB}(0,1,0)$	0.541348	$R_{AB}(-1,0,0)$	0.333993
$R_{BB}(0,0,1)$	0.59240	$R_{AB}(0,2,0)$	0.446861
$R_{BB}(1,1,0)$	0.567111	$R_{AB}(0,0,2)$	0.455289
$R_{BB}(1,0,1)$	0.600605	$R_{AB}(2,1,0)$	0.453306
$R_{BB}(2,0,0)$	0.597782	$R_{AB}(1,2,0)$	0.453306
$R_{BB}(2,1,0)$	0.604482	$R_{AB}(2,2,0)$	0.460838
$R_{BB}(1,2,0)$	0.604482	$R_{AB}(2,2,2)$	0.469071

The numerical value of this resistance is $0.333993\,R$. Some numerical values of the resistance are listed in Table 1.

3.2. *Side-centered cubic network*

The side-centered cubic lattice is a simple cubic but with additional vertices at the centers of vertical faces as shown in Fig. 2. Each unit cell consists of three vertices labeled by A, B and C.

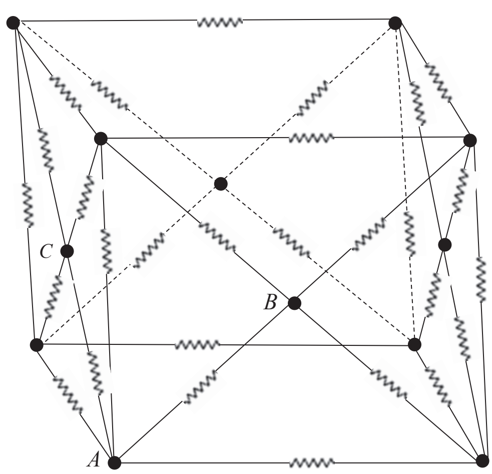

Fig. 2. The resistor network of the side-centered cubic lattice.

Applying Kirchhoff's current rule to vertex $\{\mathbf{r};\alpha\}$(with $\alpha = A, B, C$) and then using Ohm's law, the currents at vertices $\{\mathbf{r};A\}, \{\mathbf{r};B\}$ and $\{\mathbf{r};C\}$ respectively, are given by

$$\begin{aligned} RI_A(\mathbf{r}) &= \sum_{i=1}^{3}(\varphi_A(\mathbf{r}) - \varphi_A(\mathbf{r}\pm\mathbf{a}_i)) + 8\varphi_A(\mathbf{r}) - \varphi_B(\mathbf{r}) - \varphi_B(\mathbf{r}-\mathbf{a}_1) \\ &\quad - \varphi_B(\mathbf{r}-\mathbf{a}_3) - \varphi_B(\mathbf{r}-\mathbf{a}_1-\mathbf{a}_3) - \varphi_C(\mathbf{r}) - \varphi_C(\mathbf{r}-\mathbf{a}_2) \\ &\quad - \varphi_C(\mathbf{r}-\mathbf{a}_3) - \varphi_C(\mathbf{r}-\mathbf{a}_2-\mathbf{a}_3)\,, \end{aligned} \tag{18}$$

$$RI_B(\mathbf{r}) = 4\varphi_B(\mathbf{r}) - \varphi_A(\mathbf{r}) - \varphi_A(\mathbf{r}+\mathbf{a}_1) - \varphi_A(\mathbf{r}+\mathbf{a}_3) - \varphi_A(\mathbf{r}+\mathbf{a}_1+\mathbf{a}_3) \tag{19}$$

and

$$RI_C(\mathbf{r}) = 4\varphi_C(\mathbf{r}) - \varphi_A(\mathbf{r}) - \varphi_A(\mathbf{r}+\mathbf{a}_2) - \varphi_A(\mathbf{r}+\mathbf{a}_3) - \varphi_A(\mathbf{r}+\mathbf{a}_2+\mathbf{a}_3)\,. \tag{20}$$

Substituting the Fourier transforms of the electric potentials and currents given in Eqs. (2) and (3) into the above equations, the Fourier transform of the Laplacian matrix $\mathbf{L}(\theta_1,\theta_2,\theta_3)$ can be obtained as

$$\mathbf{L}(\theta_1,\theta_2,\theta_3) = \begin{pmatrix} 2\cos\theta_1 + 2\cos\theta_2 + 2\cos\theta_3 - 14 & (1+e^{-i\theta_1})(1+e^{-i\theta_3}) & (1+e^{-i\theta_2})(1+e^{-i\theta_3}) \\ (1+e^{i\theta_1})(1+e^{i\theta_3}) & -4 & 0 \\ (1+e^{i\theta_2})(1+e^{i\theta_3}) & 0 & -4 \end{pmatrix}. \tag{21}$$

The Green's function matrix $\mathbf{G}(\theta_1,\theta_2,\theta_3)$ corresponding to Fourier transform of the Laplacian matrix $\mathbf{L}(\theta_1,\theta_2,\theta_3)$ can calculated by inverting $\mathbf{L}(\theta_1,\theta_2,\theta_3)$. Because this matrix is large (elements of it are very long), we did not write it in the paper.

Again the resistance between the vertices $\{\mathbf{0};\alpha\}$ and $\{\ell_1,\ell_2,\ell_3;\beta\}$ can be evaluated by using Eq. (12). As an example, the resistance between the lattice sites $\{\mathbf{0};A\}$ and $\{1,0,0;A\}$ is given by

$$\begin{aligned} R_{AA}(1,0,0) &= \frac{R}{(2\pi)^3}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} \\ &\quad \times \frac{2(1-\cos\theta_1)d\theta_1 d\theta_2 d\theta_3}{12 - 3\cos\theta_1 - 3\cos\theta_2 - 4\cos\theta_3 - \cos\theta_1\cos\theta_3 - \cos\theta_2\cos\theta_3}\,. \end{aligned} \tag{22}$$

Performing the integral over θ_3 using residue method and then numerically for the remaining integrations, the results is $R_{AA}(1,0,0) = 0.167153\,R$. Numerical values for some additional resistances are displaced in Table 2.

Table 2. Numerical values of the effective resistance $R_{\alpha\beta}(\ell_1, \ell_2, \ell_3)$ in units of R between the vertices $\{\mathbf{0}; \alpha\}$ and $\{\ell_1, \ell_2, \ell_3; \beta\}$ in the side-centered cubic lattice.

$R_{\alpha\beta}(\ell_1, \ell_2, \ell_3)$	The values of $R_{\alpha\beta}(\ell_1, \ell_2, \ell_3)$ in units of R	$R_{\alpha\beta}(\ell_1, \ell_2, \ell_3)$	The values of $R_{\alpha\beta}(\ell_1, \ell_2, \ell_3)$ in units of R
$R_{AA}(1,0,0)$	0.167153	$R_{AB}(0,0,0)$	0.313671
$R_{AA}(0,1,0)$	0.167153	$R_{AB}(1,0,0)$	0.37669
$R_{AA}(0,0,1)$	0.156325	$R_{AB}(0,1,0)$	0.372463
$R_{AA}(1,1,0)$	0.192941	$R_{AB}(0,0,1)$	0.371377
$R_{AA}(1,0,1)$	0.18589	$R_{AB}(1,1,1)$	0.393551
$R_{AA}(0,1,1)$	0.18589	$R_{AB}(2,0,0)$	0.39594
$R_{AA}(1,1,1)$	0.198553	$R_{BC}(0,0,0)$	0.529396
$R_{AA}(2,0,0)$	0.202536	$R_{BC}(1,0,0)$	0.529396
$R_{BB}(1,0,0)$	0.531509	$R_{BC}(0,1,0)$	0.568994
$R_{BB}(0,1,0)$	0.558792	$R_{BC}(0,0,1)$	0.546919
$R_{BB}(0,0,1)$	0.528853	$R_{BC}(-1,0,0)$	0.568994
$R_{BB}(1,1,0)$	0.566881	$R_{BC}(0,1,1)$	0.572985
$R_{BB}(1,0,1)$	0.548699	$R_{BC}(1,1,1)$	0.572985
$R_{CC}(1,0,0)$	0.558792	$R_{BC}(2,0,0)$	0.568994
$R_{CC}(0,1,0)$	0.531509	$R_{BC}(0,2,0)$	0.583804
$R_{CC}(0,0,1)$	0.528853	$R_{BC}(2,1,0)$	0.579675
$R_{CC}(1,1,0)$	0.566881	$R_{BC}(2,2,0)$	0.587412
$R_{CC}(1,0,1)$	0.564986	$R_{BC}(1,2,0)$	0.583804

3.3. *Edge-centered cubic network*

The edge-centered cubic lattice is shown in Fig. 3, which is a homeomorphic expansion of simple cubic lattice with one vertex inserted on each edge. The unit cell is a cube containing four vertices labeled by $\alpha = A, B, C$ and D: A at origin (one of the corners) of the cube, and the others at the three edge-midpoints of the cube nearest the corner vertex as shown in Fig. 3. The unit cell vectors are $\mathbf{a_1}, \mathbf{a}_2$ and $\mathbf{a}_3$ along the edges of the cube.

As in previous subsections applying Kirchhoff's and Ohm's laws at vertex $\{\mathbf{r}; \alpha\}$ (with $\alpha = A,\ B,\ C$ and D), the currents are given by

$$I_A(\mathbf{r}) = \frac{\varphi_A(\mathbf{r}) - \varphi_B(\mathbf{r})}{R} + \frac{\varphi_A(\mathbf{r}) - \varphi_B(\mathbf{r} - \mathbf{a}_1)}{R} + \frac{\varphi_A(\mathbf{r}) - \varphi_C(\mathbf{r})}{R} + \frac{\varphi_A(\mathbf{r}) - \varphi_C(\mathbf{r} - \mathbf{a}_2)}{R} + \frac{\varphi_A(\mathbf{r}) - \varphi_D(\mathbf{r})}{R} + \frac{\varphi_A(\mathbf{r}) - \varphi_D(\mathbf{r} - \mathbf{a}_3)}{R}, \quad (23)$$

$$I_B(\mathbf{r}) = \frac{\varphi_B(\mathbf{r}) - \varphi_A(\mathbf{r})}{R} + \frac{\varphi_B(\mathbf{r}) - \varphi_A(\mathbf{r} + \mathbf{a}_1)}{R}, \quad (24)$$

$$I_C(\mathbf{r}) = \frac{\varphi_C(\mathbf{r}) - \varphi_A(\mathbf{r})}{R} + \frac{\varphi_C(\mathbf{r}) - \varphi_A(\mathbf{r} + \mathbf{a}_2)}{R}, \quad (25)$$

$$I_D(\mathbf{r}) = \frac{\varphi_D(\mathbf{r}) - \varphi_A(\mathbf{r})}{R} + \frac{\varphi_D(\mathbf{r}) - \varphi_A(\mathbf{r} + \mathbf{a}_3)}{R}. \quad (26)$$

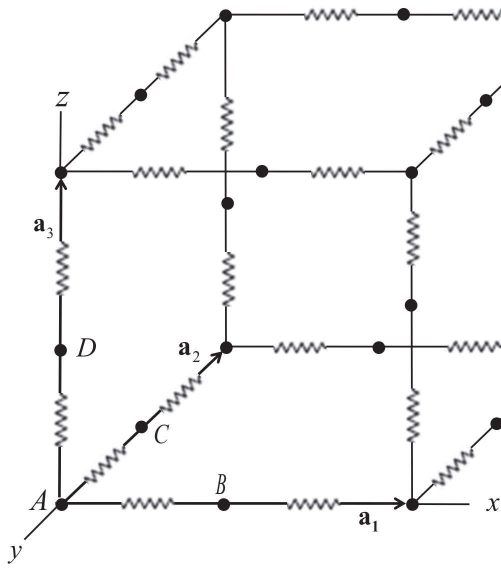

Fig. 3. The resistor network of the edge-centered cubic lattice.

From the above equations, the Fourier transform of the Laplacian matrix can be determined to be

$$\mathbf{L}(\theta_1,\theta_2,\theta_3) = \begin{bmatrix} -6 & \nu_1 & \nu_2 & \nu_3 \\ \nu_1^* & -2 & 0 & 0 \\ \nu_2^* & 0 & -2 & 0 \\ \nu_3^* & 0 & 0 & -2 \end{bmatrix} \text{ where } \nu_j = 1 + e^{-i\theta_j},\ j = 1,2,3 \qquad (27)$$

and the corresponding lattice Green's function is

$$\mathbf{G}(\theta_1,\theta_2,\theta_3) = \frac{1}{\det[\mathbf{L}(\theta_1,\theta_2,\theta_3)]} \begin{pmatrix} 8 & 4\nu_1 & 4\nu_2 & 4\nu_3 \\ 4\nu_1^* & A & 2\nu_1^*\nu_2 & 2\nu_1^*\nu_3 \\ 4\nu_2^* & 2\nu_1\nu_2^* & B & 2\nu_2^*\nu_3 \\ 4\nu_3^* & 2\nu_1\nu_3^* & 2\nu_2\nu_3^* & C \end{pmatrix} \qquad (28)$$

where

$$\det[\mathbf{L}(\theta_1,\theta_2,\theta_3)] = 8\,(3 - \cos\theta_1 - \cos\theta_2 - \cos\theta_3)\ , \qquad (29)$$

and

$$A = 16 - 4\cos\theta_2 - 4\cos\theta_3,\ B = 16 - 4\cos\theta_1 - 4\cos\theta_3 \text{ and } C = 16 - 4\cos\theta_1 - 4\cos\theta_2\ , \qquad (30)$$

Now the resistance between the nodes $\{\mathbf{0};\alpha\}$ and $\{\ell_1,\ell_2,\ell_3;\beta\}$ can be computed by using Eq. (12). However, one can note that the denominator in the resistance formula for the edge-centered cubic network obtained from Eq. (12) is the same as that in the resistance for the simple cubic network. This enables us to express

Table 3. Numerical values of the effective resistance $R_{\alpha\beta}(\ell_1, \ell_2, \ell_3)$ in units of R between the vertices $\{\mathbf{0}; \alpha\}$ and $\{\ell_1, \ell_2, \ell_3; \beta\}$ in the edge-centered cubic lattice.

$R_{\alpha\beta}(\ell_1, \ell_2, \ell_3)$	The values of $R_{\alpha\beta}(\ell_1, \ell_2, \ell_3)$ in units of R	$R_{\alpha\beta}(\ell_1, \ell_2, \ell_3)$	The values of $R_{\alpha\beta}(\ell_1, \ell_2, \ell_3)$ in units of R
$R_{AA}(1, 0, 0)$	0.666667	$R_{AB}(1, 1, 1)$	1.19317
$R_{AA}(1, 1, 0)$	0.790158	$R_{AB}(2, 0, 0)$	1.20339
$R_{AA}(1, 1, 1)$	0.836611	$R_{BC}(0, 0, 0)$	1.19754
$R_{AA}(2, 0, 0)$	0.839367	$R_{BC}(1, 0, 0)$	1.19754
$R_{AA}(2, 1, 0)$	0.867198	$R_{BC}(1, 1, 0)$	1.45751
$R_{BB}(1, 0, 0)$	1.20984	$R_{BC}(1, 0, 1)$	1.43757
$R_{BB}(1, 1, 0)$	1.44521	$R_{BC}(1, 1, 1)$	1.5109
$R_{BB}(1, 1, 1)$	1.50328	$R_{CC}(1, 0, 0)$	1.39508
$R_{AB}(0, 0, 0)$	0.666667	$R_{CC}(1, 1, 0)$	1.48005
$R_{AB}(1, 0, 0)$	1.08635	$R_{DD}(1, 0, 0)$	1.39508
$R_{AB}(0, 1, 0)$	1.06175	$R_{DD}(1, 1, 0)$	1.44521
$R_{AB}(1, 1, 0)$	1.16201	$R_{DD}(0, 1, 0)$	1.20984

the two-point resistance of the edge-centered cubic network in terms of that of the simple cubic network. For that we recalling the resistance expression for the simple cubic network given in Refs. 5 and 6, and starting from Eq. (12), we can show that the resistance on the edge-centered cubic lattice can be expressed in terms of the resistance on the simple cubic lattice. In Appendix A, the explicit expressions are summarized. Using these expressions we calculate some values of the two-point resistance and list them in Table 3. The maps between other lattices of resistors were given in Refs. 5–7.

Finally, it is very obvious from the tables that the resistance across any edge of the unit cell (cube) in the edge-centered cubic lattice is much more than that in the base and side-centered cubic lattices. The reason is the edge-centered cubic lattice provides less paths for the current flowing between the two vertices on the edge of the unit cell.

4. Conclusion

The lattice Green's function approach[7] has been used to investigate the resistance between any two vertices in infinite, decorated centered cubic lattices. The resistance on the edge-centered cubic lattice has been expressed in terms of that on the simple cubic lattice.

Appendix A. Relation between the Edge-Centered Cubic Lattice and the Simple Cubic Lattice of Resistor Networks

In this Appendix, we list the results for the resistance $R_{\alpha\beta}(\ell, m, n)$ on the edge-centered cubic lattice in terms of the resistances $R_{SC}(\ell, m, n)$ on the simple cubic lattice:

$$R_{AA}(\ell,m,n) = 2R_{SC}(\ell,m,n) \tag{A.1}$$

$$R_{BB}(\ell,m,n) = \frac{2R}{3} + 4R_{SC}(\ell,m,n) - \frac{1}{2}\{R_{SC}(\ell,m+1,n) + R_{SC}(\ell,m-1,n) + R_{SC}(\ell,m,n+1) + R_{SC}(\ell,m,n-1)\}\,, \tag{A.2}$$

$$R_{CC}(\ell,m,n) = \frac{2R}{3} + 4R_{SC}(\ell,m,n) - \frac{1}{2}\{R_{SC}(\ell+1,m,n) + R_{SC}(\ell-1,m,n) + R_{SC}(\ell,m+1,n) + R_{SC}(\ell,m-1,n)\}\,, \tag{A.3}$$

$$R_{DD}(\ell,m,n) = \frac{2R}{3} + 4R_{SC}(\ell,m,n) - \frac{1}{2}\{R_{SC}(\ell+1,m,n) + R_{SC}(\ell-1,m,n) + R_{SC}(\ell,m,n+1) + R_{SC}(\ell,m,n-1)\}\,, \tag{A.4}$$

$$R_{AB}(\ell,m,n) = \frac{R}{3} + R_{SC}(\ell,m,n) + R_{SC}(\ell+1,m,n)\,, \tag{A.5}$$

$$R_{AC}(\ell,m,n) = \frac{R}{3} + R_{SC}(\ell,m,n) + R_{SC}(\ell,m+1,n)\,, \tag{A.6}$$

$$R_{AD}(\ell,m,n) = \frac{R}{3} + R_{SC}(\ell,m,n) + R_{SC}(\ell,m,n+1)\,, \tag{A.7}$$

$$R_{BC}(\ell,m,n) = \frac{2R}{3} + \frac{1}{2}\{R_{SC}(\ell,m,n) + R_{SC}(\ell-1,m,n) + R_{SC}(\ell,m+1,n) + R_{SC}(\ell-1,m+1,n)\}\,, \tag{A.8}$$

$$R_{BD}(\ell,m,n) = \frac{2R}{3} + \frac{1}{2}\{R_{SC}(\ell,m,n) + R_{SC}(\ell-1,m,n) + R_{SC}(\ell,m,n+1) + R_{SC}(\ell-1,m,n+1)\}\,, \tag{A.9}$$

$$R_{CD}(\ell,m,n) = \frac{2R}{3} + \frac{1}{2}\{R_{SC}(\ell,m,n) + R_{SC}(\ell,m+1,n) + R_{SC}(\ell,m,n-1) + R_{SC}(\ell,m+1,n-1)\}\,. \tag{A.10}$$

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