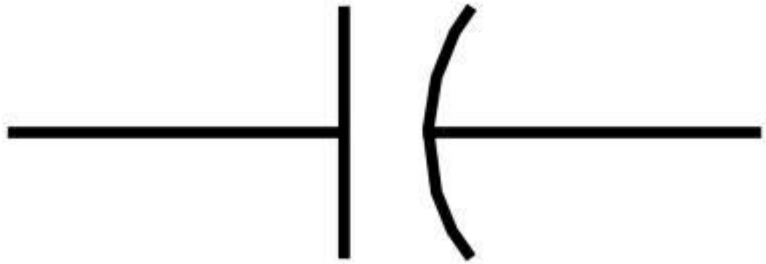
Chapter 5 – Part 2



AC – Bridges

Comparison Bridges

Capacitance Measurements

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5.5 AC - Bridges

AC - Bridges enable us to perform precise *measurements for the following* :

- **Reactance (capacitance and inductance) measurements.**
- **Determining the (*Q-factor* OR *D-factor*).**
- **Frequency measurements.**
- **Testing and analyzing of antenna and transmission line performance**

AC BRIDGE

The basic circuit of an *ac* bridge is exactly the same as the Wheatstone bridge circuit except that **impedances** are used instead of resistances, and the supply is an ***ac*-source**. Also, the null detector must be an ***ac* instrument.**

Principle

The bridge offset voltage can be found as:

$$\Delta V = \left(\frac{Z_3}{Z_1 + Z_3} - \frac{Z_4}{Z_2 + Z_4} \right) V$$

For balancing condition: (ΔV=0)

Therefore, $Z_1.Z_4 = Z_2.Z_3$

For AC-bridges, the null-condition is only valid if V_a and V_b are equal in amplitude and also same in-phase.

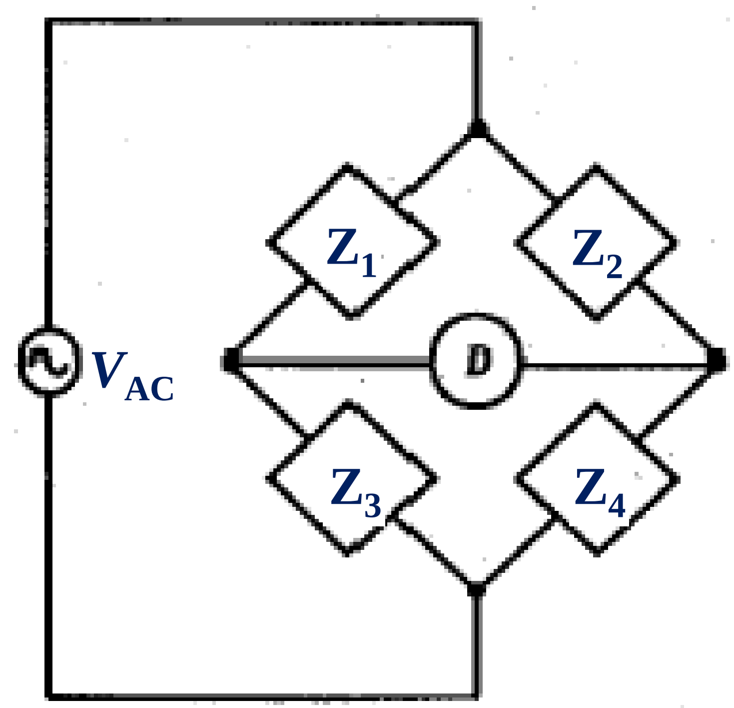
Substituting $\mathbf{Z}_i = Z_i \angle \theta_i$,

$$Z_2 Z_3 \angle(\theta_2 + \theta_3) = Z_1 Z_4 \angle(\theta_1 + \theta_4)$$

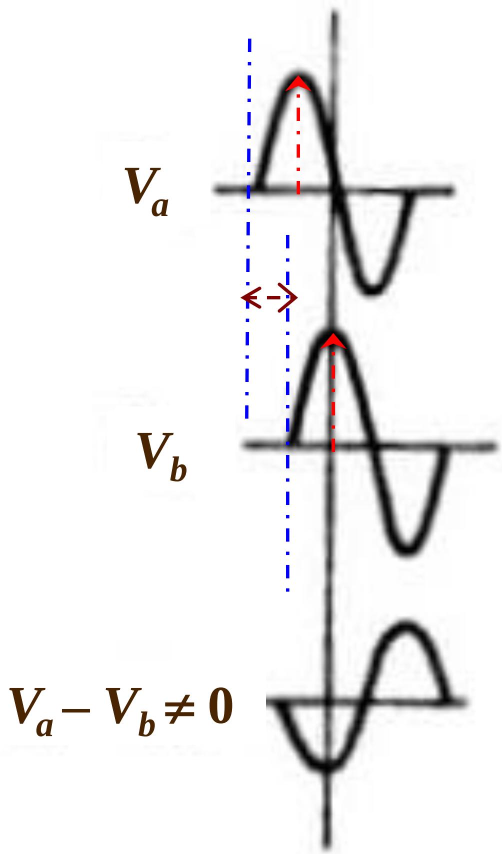
* **First balance condition** $\Rightarrow$ $Z_2 Z_3 = Z_1 Z_4$ **=amplitude**

* **2nd balance condition** $\Rightarrow$ $\angle(\theta_2 + \theta_3) = \angle(\theta_1 + \theta_4)$ **in-phase**

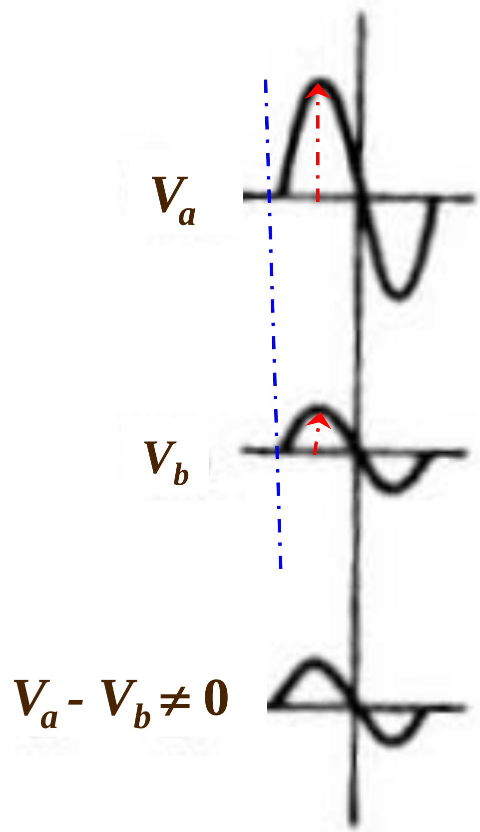
Structure



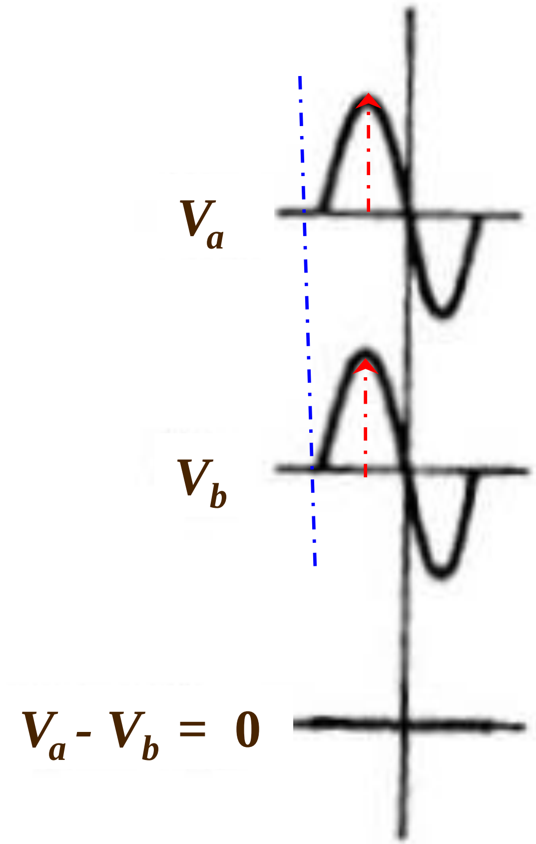
EXAMPLE : V_A AND V_B



V_a and V_b are equal in amplitude but not in-phase.



V_a and V_b are equal in-phase but not equal in amplitude.



V_a and V_b are equal in amplitude and in-phase.

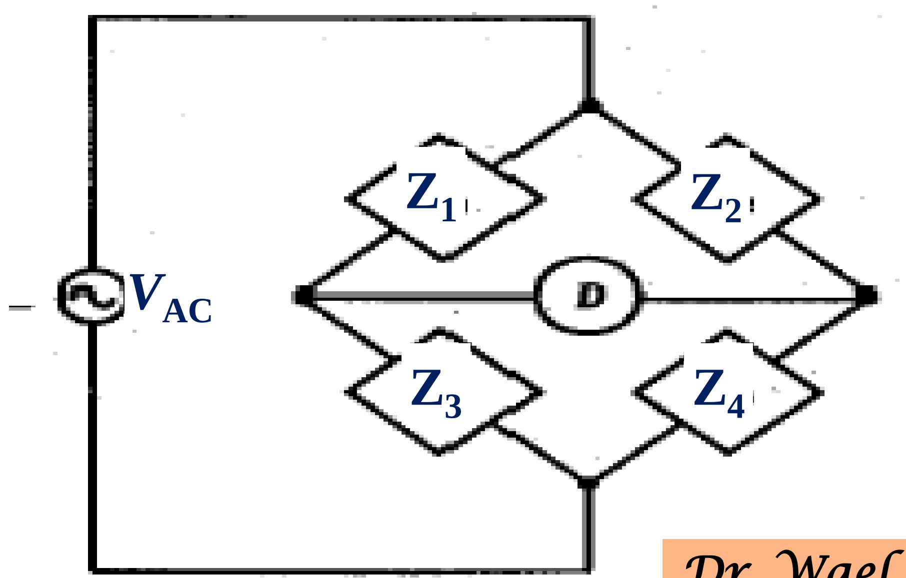
Example 7

The impedances of the bridge arms are

$Z_1 = 200\Omega\angle 30°$; $Z_2 = 150\Omega\angle 0°$

$Z_3 = 250\Omega\angle -40°$; $Z_x = Z_4 = \text{unknown}$

Determine the values of the unknown arm.



Solution 7

The first condition for balance requires that

$$Z_1 Z_x = Z_2 Z_3 \; ; \qquad \therefore Z_x = \frac{Z_2 Z_3}{Z_1} = \frac{150\Omega \times 250\Omega}{200\Omega} = 187.5\Omega$$

The second condition for balance requires that the sum of the phase angles of opposite arms should be equal

$$\angle\phi_1 + \angle\phi_x = \angle\phi_2 + \angle\phi_3 \; ; \qquad \therefore \phi_x = \phi_2 + \phi_3 - \phi_1 = 0° + (-40°) - 30° = -70°$$

Hence the unknown arm impedance Z_x can be written as

$$Z_x = 187.5\Omega\angle -70° \;=\; (64.13 - j176.19)\Omega$$

Where, $Z_x = x + j\,y =$ ***R + j X***

$x = r\cos\varphi = 187.5\cos(-70) =$ **64.13**

$y = r\sin\varphi = 187.5\sin(-70) =$ **-176.19**

Capacitors equivalent circuits

Capacitor Equivalent Circuits:

The equivalent circuit of a capacitor consists of a pure capacitance C and a resistance R .

Where, C_p represents the actual capacitance value,

and R_p represents the resistance of the ***dielectric*** or ***leakage resistance***.

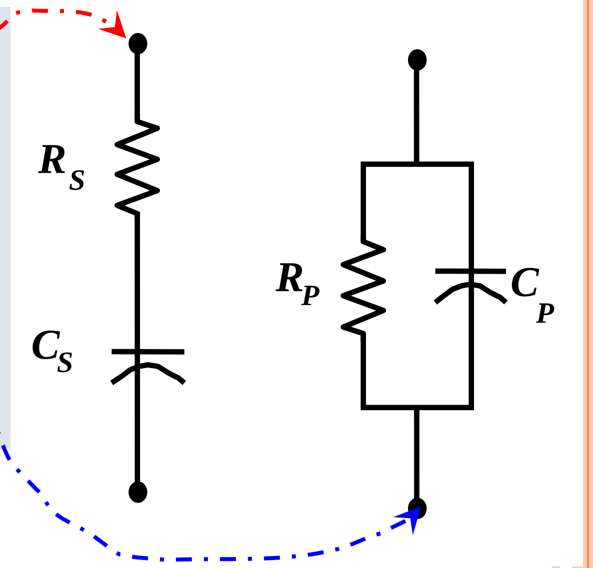
- **Capacitors with a high-resistance dielectric are best represented by a series RC circuit,**
- **Capacitors with a low-resistance dielectric represented by the parallel RC circuit .**

The series impedance is

$$Z_S = R_S - \frac{j}{wC_S}$$

The parallel admittance is

$$Y_p = \frac{1}{R_p} + j\,wC_p$$



D-FACTOR OF A CAPACITOR

The quality of a capacitor can be expressed in **terms of its power dissipation.**

A very pure capacitor has a high dielectric resistance (low leakage current) and virtually zero power dissipation.

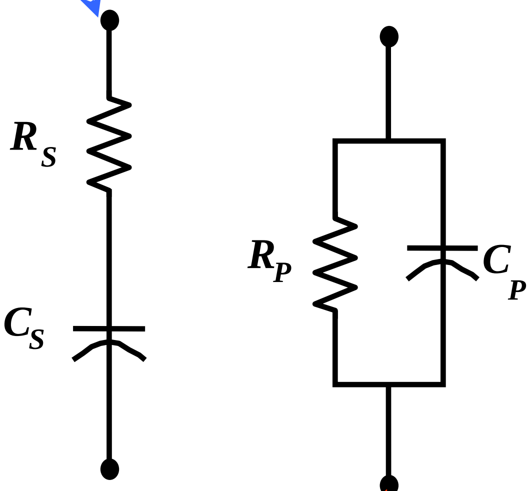
The ***dissipation factor*** or ***D-factor*** is simply the ***ratio*** of the **component reactance** (at a given frequency) to **the resistance** measurable at its terminals.

For a parallel equivalent circuit

$$D = \frac{X_p}{R_p} = \frac{1}{wC_pR_p}$$

For a series equivalent circuit

$$D = \frac{R_s}{X_s} = wC_sR_s$$



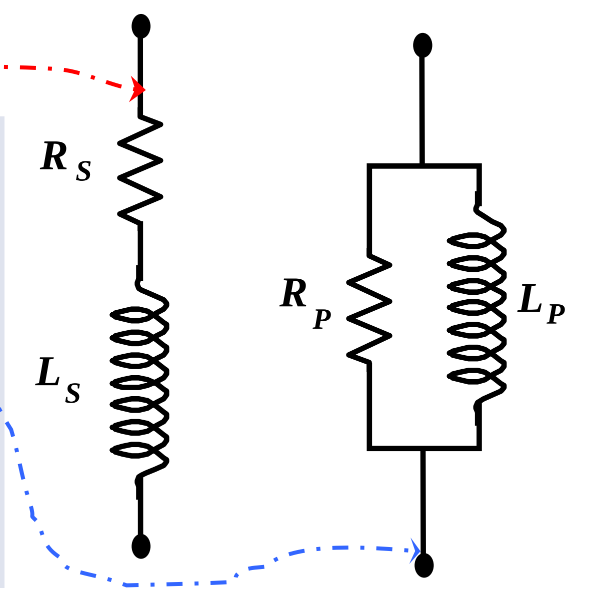
INDUCTORS EQUIVALENT CIRCUITS

Inductor Equivalent Circuits:

- **The series equivalent circuit represents an inductor as a pure inductance L_S, in series with the resistance of its coil R_S.**
- **This series equivalent circuit is normally the best way to represent an inductor, because the actual winding resistance is involved and this is an important quantity.**
- **The parallel *RL* equivalent circuit for an inductor can also be used.**

The series impedance is $Z_s = R_s + jwL_s$

The parallel admittance is $Y_p = \frac{1}{R_p} - \frac{j}{wL_p}$

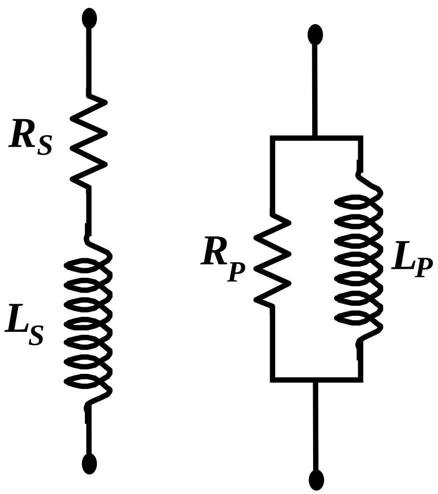


Q - Factor of an Inductor

The quality of an inductor can be defined in terms of its power dissipation.

An ideal inductor should have zero winding resistance, and therefore zero power dissipated in the winding.

The *quality factor* or *Q-factor*, of the inductor is the *ratio* of its inductive reactance to its resistance at the op [illegible]**.**



For a series equivalent circuit: $$Q = \frac{X_s}{R_s} = \frac{wL_s}{R_s}$$

For a parallel equivalent circuit : $$Q = \frac{R_p}{X_p} = \frac{R_p}{wL_p}$$

5.6.1. Capacitance Comparison Bridges

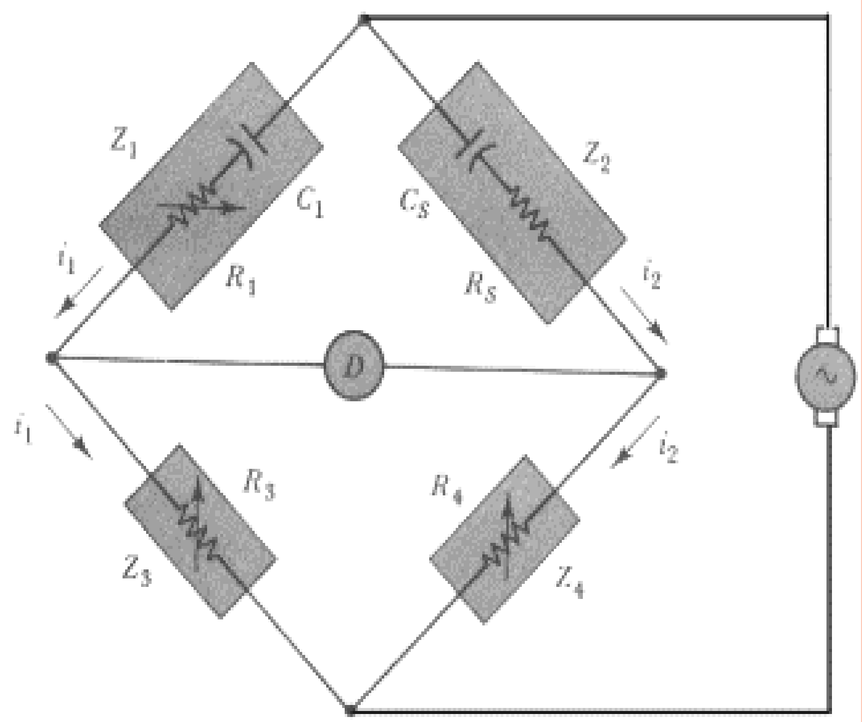
i) Series-Resistance Capacitance Bridge (also known as Similar-angle bridge)

- **The unknown capacitance C_S is represented in series with resistance R_S.**
- **A standard adjustable resistance R_1 is connected in series with C_1.**
- **To obtain a bridge balance, resistors R_1 and either C_1 are adjusted alternately.**

The impedances are:

$$Z_1 = R_1 - \frac{j}{wC_1} \quad ; \quad Z_3 = R_3$$

$$Z_2 = R_s - \frac{j}{wC_s} \quad ; \quad Z_4 = R_4$$



When the bridge is balanced:

$$Z_1 \cdot Z_4 = Z_2 \cdot Z_3$$

$$\left[R_1 - \frac{j}{wC_1}\right] \cdot R_4 = \left[R_s - \frac{j}{wC_s}\right] \cdot R_3$$

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1. Capacitance Bridges

Then the equation is simplified as:

$$R_1 R_4 - j\frac{R_4}{wC_1} = R_s R_3 - j\frac{R_3}{wC_s}$$

After equating the real terms for both sides, we get:

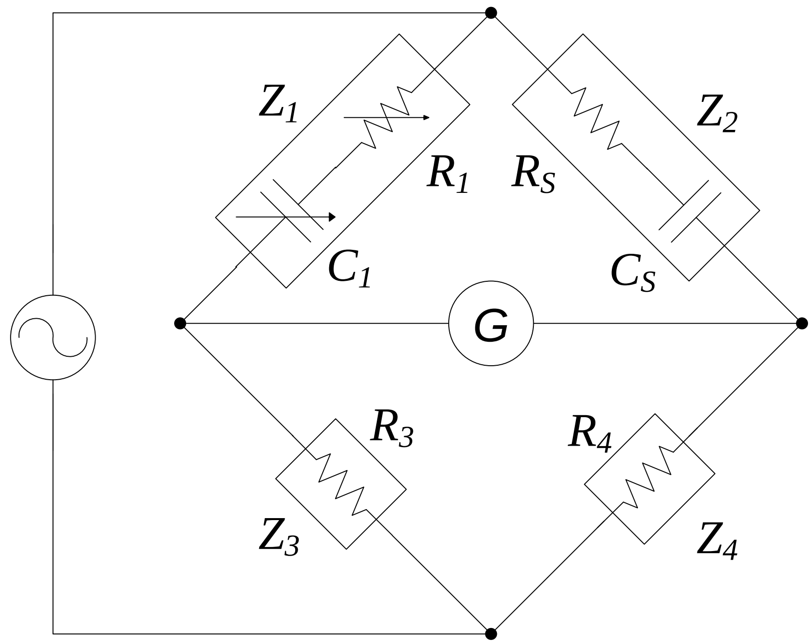
$$R_1 R_4 = R_s R_3 \quad \Longrightarrow \quad R_S = \frac{R_1 R_4}{R_3}$$

And by equating the Imaginary terms both sides, we get: :

$$j\frac{R_4}{wC_1} = j\frac{R_3}{wC_s} \quad \Longrightarrow \quad C_S = \frac{C_1 R_3}{R_4}$$

Example

A series resistance-capacitance bridge as shown below has a $0.1\mu F$ standard capacitor for $C1$, and $R3$= 10kΩ. Balance is achieved with 100Hz supply frequency when $R1$=125Ω and $R4$=14.7kΩ. Calculate the ***resistive*** and ***capacitive*** components of measured capacitor and its ***dissipation factor***.



Solution

$$C_S = \frac{C_1 R_3}{R_4} = \frac{0.1\mu \times 10k}{14.7k} = 0.068\mu F$$

$$R_S = \frac{R_1 R_4}{R_3} = \frac{125 \times 14.7k}{10k} = 183.75\Omega$$

$$D = wC_S R_S = 2\pi \times 100 \times 0.068 \times 10^{-6} \times 183.75 = 0.00785$$

II) Parallel-Resistance Capacitance Bridge (also known as Similar-angle bridge)

In this bridge the unknown capacitance is represented by its parallel equivalent circuit; C_p in parallel with R_p, and Z_3 and Z_4 are resistors.

- **The bridge balance is achieved by adjustment of R_1 and C_1.**

The load impedances are:

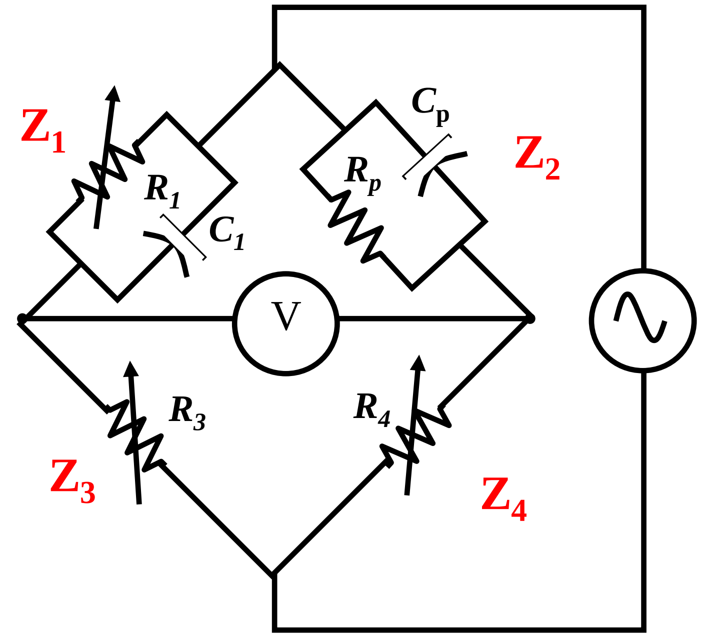
$$Y_1 = \frac{1}{Z_1} = \frac{1}{R_1} + jwC_1 \quad ; \quad Y_p = \frac{1}{Z_2} = \frac{1}{R_p} + jwC_p$$

$$Z_3 = R_3 \quad ; \quad Z_4 = R_4$$

When the bridge is balanced:

$$Z_1 Z_4 = Z_2 Z_3$$

$$\left(\frac{R_1}{1 + jwC_1R_1}\right) \cdot R_4 = \left(\frac{R_p}{1 + jwC_pR_p}\right) \cdot R_3$$



After equating the real & Imaginary terms:

$$R_P = \frac{R_1 R_4}{R_3} \quad \text{and} \quad C_P = \frac{C_1 R_3}{R_4}$$

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EXERCISE 6

Find the parallel equivalent circuit for C_S and R_S values determined in Example 8. Also determine the component values of R_1 and R_4 required to balance the calculated C_P and R_P values in a parallel resistance-capacitance bridge. Assume that R_3 remains at 10kΩ and C_1 at $0.1\mu F$.

Solution Exercise 6

Example

A parallel-resistance capacitance bridge has a standard capacitance value of **0.1μF.** Balance is achieved at a supply frequency of **100Hz** when R_3**= 10kΩ**, R_1**=375kΩ** , and R_4**=14.7kΩ**.

Determine the resistive and capacitive components of the measured capacitor and its dissipation factor (***D-factor***).

Solution

For the given parallel-resistance capacitance bridge, find R_p, C_p and the *D*-factor :-

$$C_P = \frac{C_1 R_3}{R_4} = \frac{0.1\mu F \times 10k\Omega}{14.7k\Omega} = 0.068\mu F$$

$$R_P = \frac{R_1 R_4}{R_3} = \frac{375k\Omega \times 14.7k\Omega}{10k\Omega} = 551.3k\Omega$$

The dissipation factor is the *D-factor*

$$D = \frac{1}{wC_P R_P} = \frac{1}{2\pi \times 100 \times 0.068 \times 10^{-6} \times 551.3 \times 10^{3}} \approx 42.5 \times 10^{-3}$$