CHAPTER 5

BRIDGES AND THEIR APPLICATION

Resistance Measurements

Dr. Wael Salah
RESISTANCE MEASUREMENTS

Conventional Ways of Measuring Resistance:-

1) Using a **Ohmmeter** – Convenient but inaccurate, requires calibration prior to measurement.

2) Using the **Ammeter** and **Voltmeter** method - i.e. by measuring the current and voltage across the unknown resistance, then calculating the value of the resistance using Ohms Law. This method not only requires calibration prior to measurement, it also introduces error by adding the ammeter and voltmeter elements into the measurement.

3) **Bridge Circuits** Measuring Resistance; to be introduced in this chapter.
The voltmeter measures the voltage $E$ across the resistor $R$, but the ammeter indicates the resistor current $I$ plus the voltmeter current $I_v$.  

$$R = \frac{E}{I + I_v}$$ 

(a) Voltmeter connected across load
In this configuration, the ammeter is directly in series with resistor $R$. In this case the voltmeter indicates the resistor voltage $E$ plus the ammeter voltage drop $E_A$.

\[ R = \frac{E + E_A}{I} \]
Bridge Circuits

Bridge circuits, which are instruments for making comparative measurements, are widely used to measure resistance, inductance, and capacitance.

Bridge circuits operate on a null-indication principle, which means the measurement is independent of calibration, this makes the measurement device very accurate.

Bridge circuits are also frequently used in control circuits.

When used in such applications, one arm of the bridge should contain a resistive element that is sensitive to the physical parameter (Such as: temperature, pressure, etc.) being controlled.

Dr. Wael Salah
Advantages of the bridges:

- Most accurate types of measurement devices.
- Reliable and very easy to perform.
- Small in size and low power operation.
- Less maintenance required.

Disadvantages of the bridges:

- To ensure the highest accuracy of the bridge circuit, the detector need to be extremely sensitive.
TYPES OF BRIDGE CIRCUITS

DC-Bridges
• Wheatstone Bridge (Varying Resistance to null bridge)
• Current Balance Bridge (Varying Current to null bridge)
• Kelvin Bridge (Measurement of Low Resistance)

AC-Bridges
• Capacitive and inductive bridges.
• Maxwell & Wien Bridge
• Hay Bridge
• Anderson Bridge
• Radio Frequency Bridge
• Schering Bridge
DC BRIDGES – WHEATSTONE BRIDGE

Common name for a normal DC bridge is (Wheatstone Bridge).

- Introduced by Sir Charles Wheatstone as early as 1847.
- The Wheatstone bridge consists of two parallel resistance branches each branch containing two series elements, usually resistors.
- A dc voltage source is connected across this resistance network to provide a source of current through the resistance network.
- A null-detector, usually a galvanometer, is connected between the parallel branches to detect a condition of balance.

- The Wheatstone bridge has been in use longer than almost any other electrical measuring instrument.
- It is still an accurate and reliable instrument and is heavily used in industry. Accuracy of 0.1% is quite common with Wheatstone as opposed to 3% to 5% error with the ordinary ohmmeters for resistance measurement.
What is a Galvanometer?

Galvanometer is the historical name given to a **moving coil electric** current detector. When a current is passed through a coil in a magnetic field, the coil experiences a torque proportional to the current. If the coil's movement is opposed by a coil spring, then the amount of deflection of a needle attached to the coil may be proportional to the current passing through the coil.
The Wheatstone Bridge

- It is the traditional instrument used for making accurate measurements of impedance (resistance, reactance, or both).
- Suitable for moderate resistance values: $1 \, \Omega$ to $10 \, M\Omega$
- Balance condition, when no potential difference across the galvanometer ($V_{DB}=0$)

**null-indication**

- Under this condition
  \[
  V_{AD} = V_{AB}, \quad I_1 R_1 = I_2 R_2, \quad I_3 R_3 = I_4 R_x
  \]
  
  Ratio $(R_2/R_1)$ called multiplier

- When current in galvanometer is 0, so $I_1=I_3$ & $I_2=I_4$

  \[
  \frac{R_1}{R_3} = \frac{R_2}{R_x} \quad R_x = R_3 \frac{R_2}{R_1}
  \]
DC BRIDGES - WHEATSTONE BRIDGE

Voltage Potential and Current Movement

The **difference** in potential is crucial for current flow—not the value of the potential to ground of the end points.
\[ I_1 = V \div R = 12V \div (10\Omega + 20\Omega) = 0.4A \]

\[ V_{R2} = I \times R_2 = 0.4A \times 20\Omega = 8 \text{ volts} \]

\[ V_{R1} = 4V \text{ and } V_{R2} = 8V \]

both points have the same value of 8 volts: \( C = D = 8 \text{ volts} \)

the difference is: \( 0 \text{ volts} \)

When this happens, both sides of the parallel network are said to be \textbf{balanced} because the voltage at point C is the same value as the voltage at point D.
Consider what would happen when we reverse the position of the two resistors, $R_3$ and $R_4$ in the second parallel branch.

![Circuit Diagram]

$$V_{R4} = 0.4A \times 10\Omega = 4 \text{ volts}$$

The voltage difference between points C and D will be 4V as: $C = 8V$ and $D = 4V$

When this happens the parallel network is said to be **unbalanced** as the voltage at point C is at a different value to the voltage at point D.
DC BRIDGES - WHEATSTONE BRIDGE

The bridge consists of an unknown resistance (to be measured) $R$, two precision resistors $P$ and $Q$, an adjustable resistor $S$.

To determine $R$, the variable resistance $S$ is adjusted until the galvanometer indicates zero voltage.

Thus, the bridge is said to be balanced,

**When:** Galvanometer indicates $\Rightarrow 0V$,

\[ V_P = V_Q \quad \text{and} \quad V_R = V_S \]

Since $I_G = 0$ ;

\[ I_1P = I_2Q \quad \text{and} \quad I_1R = I_2S \]

Dividing the 2 equations : \[ \frac{I_1R}{I_1P} = \frac{I_2S}{I_2Q} \]

Therefore, the unknown resistance ($R$) :

\[ R = \frac{S.P}{Q} \]

Dr. Wael Salah
DC BRIDGES - WHEATSTONE BRIDGE

**Example 1:** Determine the value of the unknown resistor $R_X$. Assuming the circuit is balanced at $R_1 = 12k\Omega$, $R_2 = 15k\Omega$ and $R_3 = 32k\Omega$

\[
R_X = \frac{R_2 \cdot R_3}{R_1} = \frac{15 \times 32}{12} = 40k\Omega
\]

*Answer:*

$R_X = 40k\Omega$
**Exercise 1**
A Wheatstone bridge has $P = 5.5\, \Omega$, $Q = 7\, \Omega$ and galvanometer $G = 0\, \text{V}$ when $S = 5.1\, \Omega$.

a) Calculate the value of $R$

b) Determine the resistance measurement range for the bridge if $S$ is adjustable from $1\, \Omega$ to $8\, \Omega$.

$$R = \frac{SP}{Q}$$
DC BRIDGES - WHEATSTONE BRIDGE

Solution Exercise 1

a) \( P = 5.5\,k\Omega, \; Q = 7\,k\Omega, \; G = 0\,V \) when \( S = 5.1\,k\Omega \).

\[
R = \frac{S \cdot P}{Q}
\]

b) 

\( S = 1\,k\Omega \)

\( S = 8\,k\Omega \)
Measurement Errors in the Wheatstone Bridge

The Wheatstone bridge is mainly used for low value resistance measurements.

1. Error in the Wheatstone bridge is found due to limiting errors of the resistors.
2. Insufficient sensitivity of null detector.
3. Change of resistance due to heating effect.
4. Thermal emf in the Bridge or Galvanometer
5. Error due to the resistance of leads and circuit contacts.
PARAMETER OF THE WHEATSTONE BRIDGE

1) Sensitivity of the Wheatstone Bridge
2) Voltage Offset
3) Offset Current
PARAMETER OF THE WHEATSTONE BRIDGE

1) Sensitivity of the Wheatstone Bridge

- When the bridge is in an unbalanced condition, current flows through the galvanometer, causing a deflection of its pointer.

- The amount of deflection is a function of the sensitivity of the galvanometer.

- The sensitivity is deflection per unit current.

\[ S = \frac{\text{milimeters}}{\mu A} = \frac{\text{degrees}}{\mu A} = \frac{\text{radian}}{\mu A} \]

- The more sensitive the galvanometer will deflect more with the same amount of current.

Therefore, it follows that total deflection \( D \) is

\[ D = SI \]

where \( S \) is as defined and \( I \) is the current in microamperes (\( \mu A \)).
THEVENIN EQUIVALENT CIRCUIT

It is necessary to calculate the galvanometer circuit -- to determine whether or not the galvanometer has the required sensitivity to detect an unbalance conditions.

Different galvanometer not only may require different currents per unit deflection (current sensitivity), but also may have a difference internal resistance.

✓ The deflection current in the galvanometer is,

\[ I_g = \frac{E_{th}}{R_{th} + R_g} \]

✓ \( R_g \) = the internal resistance in the galvanometer
Thevenin Equivalent Circuit

Converting the Wheatstone bridge to its Thevenin equivalent circuit in order to find the current follows in the galvanometer: There are two steps must be taken:

- Finding the **equivalent voltage** when the galvanometer is removed from the circuit (the open voltage between A and B of bridge).
- Finding the **equivalent resistance**, with the battery replaced by its internal resistance (removing the voltage source and makes its side short circuit and removing current source makes its side open circuit)
THEVENIN EQUIVALENT CIRCUIT

Calculate $V_{th}$

$$V_{th} = V_1 - V_2 = I_1 R_1 - I_2 R_2$$

$$V_{th} = \frac{E}{R_1 + R_3} R_1 - \frac{E}{R_2 + R_4} R_2$$

$$= E \left( \frac{R_1}{R_1 + R_3} - \frac{R_2}{R_2 + R_4} \right)$$

Calculate $R_{th}$

$$r_{th} = \frac{R_1 R_3}{R_1 + R_3} + \frac{R_2 R_4}{R_2 + R_4}$$

$$R_{Th} = R_{ab} = \left( \frac{R_1}{R_3} \right) + \left( \frac{R_2}{R_4} \right)$$

Dr. Wael Salah
Example
Calculate the current passes in the galvanometer of the following circuit.

Solution:

1. Find $V_{th}$

$$V_{th} = E \left( \frac{R_1}{R_1 + R_3} - \frac{R_2}{R_2 + R_4} \right)$$

$$V_{th} = 6 \times \left( \frac{1\,k\Omega}{1\,k\Omega + 3.5\,k\Omega} - \frac{1.6\,k\Omega}{1.6\,k\Omega + 7.5\,k\Omega} \right) = 0.278\,V$$

2. Find $r_{th}$

$$r_{th} = \frac{R_1 R_3}{R_1 + R_3} + \frac{R_2 R_4}{R_2 + R_4}$$

$$r_{th} = \frac{1\,k\Omega \times 3.5\,k\Omega}{1\,k\Omega + 3.5\,k\Omega} + \frac{1.6\,k\Omega \times 7.5\,k\Omega}{1.6\,k\Omega + 7.5\,k\Omega} = 2.096\,k\Omega$$

3. Find $I_G$

$$I_G = \frac{V_{th}}{r_{th} + R_G} = \frac{0.278\,V}{2.096 \times 10^3\,\Omega + 200\,\Omega} = 121.4\,\mu A$$
PARAMETER OF THE WHEATSTONE BRIDGE

2) Voltage Offset

If we assume the galvanometer impedance is infinite, which is an open circuit.

If the bridge is balanced, then:

$$\Delta V = 0 \text{ and } V_a = V_b$$

If the bridge is unbalanced, then the potential difference is:

$$\Delta V \neq 0 = V_a - V_b = V \cdot \frac{R_3}{R_1 + R_3} - V \cdot \frac{R_4}{R_2 + R_4} = V \left[ \frac{R_3}{R_1 + R_3} - \frac{R_4}{R_2 + R_4} \right]$$
A bridge circuit in Figure below is used for potential measurement nulls when \( R_1 = R_2 = 1\,k\Omega \), \( R_3 = 605\,\Omega \), and \( R_4 = 500\,\Omega \) with a 10V supply. **Find the offset voltage.**

\[
\Delta V \neq 0 = V_a - V_b = V \cdot \frac{R_3}{R_1 + R_3} - V \cdot \frac{R_4}{R_2 + R_4} = V \cdot \left[ \frac{R_3}{R_1 + R_3} - \frac{R_4}{R_2 + R_4} \right]
\]
PARAMETER OF THE WHEATSTONE BRIDGE

Solution Exercise 2
Parameter of the Wheatstone Bridge

3) Offset Current

Offset current is the current drawn by the galvanometer when the bridge is unbalanced.

The easiest way to determine this offset current is to find the Thevenin equivalent circuit between point \( a \) and \( b \) of the bridge.

The Thevenin voltage and resistance are the voltage and resistance measured between point \( a \) and \( b \), so
The Thevenin voltage and resistance are calculated for the *unbalanced* bridge as below:

\[ V_{Th} = \Delta V = \frac{R_3}{R_1 + R_3} V - \frac{R_4}{R_2 + R_4} V \]

\[ R_{th} = R_1 \parallel R_3 + R_2 \parallel R_4 \]

And the *offset current* can be calculated:

\[ I_G = \frac{V_{th}}{R_G + R_{th}} \]

Hence, \( I_G \neq 0 \)
Example 2:
A bridge circuit has resistance of \( R_1 = R_2 = R_3 = 2.5k\Omega \) and \( R_4 = 2k\Omega \) and a 15V supply. If galvanometer with a 50\( \Omega \) internal resistance is used for a detector, **find the offset current**.

**Solution**

\[
V_{Th} = \frac{R_2 R_3 - R_1 R_4}{(R_1 + R_3)(R_2 + R_4)} V = \frac{2.5k \times 2.5k - 2.5k \times 2k}{(2.5k + 2.5k)(2.5k + 2k)} 15 = 0.833V
\]

\[
R_{Th} = \frac{R_1 R_3}{R_1 + R_3} + \frac{R_2 R_4}{R_2 + R_4} = \frac{2.5k \times 2.5k}{2.5k + 2.5k} + \frac{2.5k \times 2k}{2.5k + 2k} = 2.361k\Omega
\]

The offset current:

\[
I_G = \frac{V_{Th}}{R_G + R_{Th}} = \frac{0.833}{50 + 2.361k} = 3.45 \times 10^{-4} A
\]
BALANCED BRIDGE USEFUL ELECTRONICS
APPLICATIONS

• Used to measure changes in light intensity, pressure or strain.
• The types of resistive sensors that can be used within a wheatstone bridge circuit include:
  ✓ Photoresistive sensors (LDR’s)
  ✓ Positional sensors (potentiometers),
  ✓ Piezoresistive sensors (strain gauges) and
  ✓ Temperature sensors (thermistor’s), etc.

Dr. Wael Salah
Application: Wheatstone Bridge Light Detector

One of the resistors within the bridge network is replaced by a light dependent resistor, or LDR.

An LDR, also known as a cadmium-sulphide (Cds) photocell, is a passive resistive sensor which converts changes in visible light levels into a change in resistance and hence a voltage.

Light dependent resistors can be used for monitoring and measuring the level of light intensity, or whether a light source is ON or OFF.
**APPLICATION: STRAIN GAUGE**

Gauge factor, $G_F$

$$G_F = \frac{\Delta R / R_0}{\Delta L / L} = \frac{\Delta R / R_0}{\varepsilon}$$

$$R_3 = R_0 + \Delta R = R_0 \left(1 + \frac{\Delta R}{R_0}\right)$$

$$R_3 = R_0 \left(1 + G_F \varepsilon\right)$$

**Dr. Wael Salah**
APPLICATION: MEASURING STRAIN

\[ V_0 = V_{dc} \left[ \frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4} \right] \]

\[ V_0 = V_{dc} \left[ \frac{R_2}{R_1 + R_2} - \frac{R_4}{R_0 (1 + G_F \varepsilon) + R_4} \right] \]

Solving for \( \varepsilon \)

\[ \varepsilon = \frac{R_4}{G_F R_0} \left[ \frac{1}{\left( \frac{R_2}{R_1 + R_2} - \frac{V_0}{V_{dc}} \right)} - 1 \right] - \frac{1}{G_F} \]

\[ R_3 = R_0 (1 + G_F \varepsilon) \]
Wheatstone Bridge Limitation

- It is often necessary to make measurements of low resistances, such as for samples of wires or low values of meter-shunt resistors.

- Resistors in the range of approximately $1\mu\Omega$ to $1\Omega$ may be measured with a high degree of accuracy using a bridge called the Kelvin bridge.

- Kelvin bridge is a modified version of the Wheatstone bridge. The purpose of the modification is to eliminate the effects of contact, and lead resistance when measuring unknown low resistances.
The connecting leads in a Wheatstone bridge circuit can introduce errors when measuring very low resistances. The $Y$ resistance could be taken as part of $Q$ or part of $S$. 

Dr. Wael Salah
Four Terminal Resistors

Four-terminal resistors have current terminals and voltage terminals. The resistance is defined as that between the voltage terminals, so that there is no error introduced by contact voltage drops at the current terminals.
Kelvin’s Bridge

In the Wheatstone bridge, the bridge contact and lead resistance causes significant error, while measuring low resistances. Thus for measuring the values of resistance below 1Ω, the modified form of Wheatstone bridge is used, known as Kelvin bridge. The consideration of the effect of contact and lead resistances is the basic aim of the Kelvin bridge.

The Fig. is the basic circuit of the Kelvin bridge.

Dr. Wael Salah
KELVIN’S BRIDGE

The resistance $R_y$ represents the resistance of the connecting leads from $R_3$ to $R_x$. The resistance $R_x$ is the unknown resistance to be measured.

The galvanometer can be connected to either terminal a, b or terminal c. When it is connected to a, the lead resistance $R_y$ gets added to $R_x$ hence the value measured by the bridge, indicates much higher value of $R_x$.

If the galvanometer is connected to terminal c, then $R_y$ gets added to $R_3$. This results in the measurement of $R_x$ much lower than the actual value.

The point b is in between the points a and c, in such a way that the ratio of the resistance from c to b and that from a to b is equal to the ratio of $R_1$ and $R_2$.

\[
\therefore \quad \frac{R_{cb}}{R_{ab}} = \frac{R_1}{R_2}
\]

Now the bridge balance equation in its standard form is,

\[
R_1R_3 = R_2R_x
\]
But \( R_3 \) and \( R_x \) now are changed to \( R_3 + R_{ab} \) and \( R_x + R_{cb} \) respectively due to lead resistance.

\[
\therefore \quad R_1 \left( R_3 + R_{ab} \right) = R_2 \left( R_x + R_{cb} \right) \quad \ldots (3)
\]

\[
\therefore \quad (R_x + R_{cb}) = \frac{R_1}{R_2} \left( R_3 + R_{ab} \right) \quad \ldots (4)
\]

Now we have, \( \frac{R_{cb}}{R_{ab}} = \frac{R_1}{R_2} \)

\[
\therefore \quad \frac{R_{cb}}{R_{ab}} + 1 = \frac{R_1}{R_2} + 1 \quad \ldots \text{adding 1 to both sides}
\]

\[
\frac{R_{cb} + R_{ab}}{R_{ab}} = \frac{R_1 + R_2}{R_2} \quad \ldots (5)
\]

But \( R_{cb} + R_{ab} = R_y \)

Substituting in (5) we get,

\[
\frac{R_y}{R_{ab}} = \frac{R_1 + R_2}{R_2} \quad \ldots (6)
\]

\[
\therefore \quad R_{ab} = \frac{R_2 R_y}{R_1 + R_2} \quad \ldots (7)
\]
This concludes that the effect of the resistance of the connecting lead a to c has been eliminated by connecting the galvanometer at the intermediate point c.
The Kelvin bridge is essentially a Wheatstone bridge with two additional resistors (A and B). These are included to eliminate the voltage drop across Y from the balance equation.
Kelvin’s Double Bridge

Figure 8-8 Kelvin bridge for very low resistance measurement. $S$ is a standard four-terminal resistor, $Q$ is the resistor to be measured, and $P$, $R$, $A$, and $B$ are precision resistors. When the bridge is balanced, $Q = S \frac{P}{R}$. 
DC BRIDGES – Kelvin Double Bridge

- It is used to solve the problem of connecting leads
- it has two balanced ratio
- can measure small resistance ($0.0001\Omega$) with error 0.1%

The balance conditions are:

- $V_{lk} = V_{imp}$
- $V_{ok} = V_{onp}$
\[ V_{IK} = V \frac{R_1}{R_2 + R_1} \]  

\[ V_{lo} = I[R_3 + R_x + (R_b + R_a) // R_y] \]  

\[ I_{R_Y} R_y = I_{mpn}(R_b + R_a) \]  

\[ V_{imp} = IR_3 + I * \frac{R_y}{R_a + R_b + R_y} R_b \]  

From Eqs. (1)-(2) and rearrange, we have

\[ R_x = R_3 \frac{R_2}{R_1} + \frac{R_b R_y}{R_a + R_b + R_y} \left( \frac{R_2}{R_1} - \frac{R_a}{R_b} \right) \]  

With the balance condition \( \frac{R_a}{R_b} = \frac{R_2}{R_1} \)

\[ R_x = R_3 \frac{R_2}{R_1} \]