Chapter 5

Bridges and their Application

Resistance Measurements

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Resistance Measurements

Conventional Ways of Measuring Resistance:-

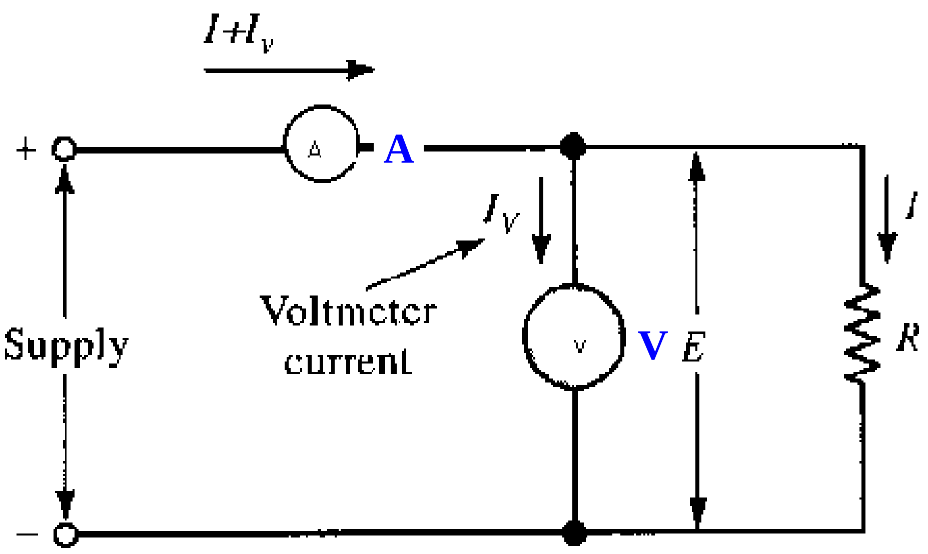
1) Using a **Ohmmeter** – Convenient but inaccurate, requires calibration prior to measurement.
2) Using the **Ammeter** and **Voltmeter** method -i.e. by measuring the current and voltage across the unknown resistance, then calculating the value of the resistance using Ohms Law. This method not only requires calibration prior to measurement, **it also introduces error by adding** the ammeter and voltmeter elements into the measurement.

3) Bridge Circuits Measuring Resistance; to be introduced in this chapter.

Resistance Measurements

The voltmeter measures the voltage ***E*** across the resistor ***R***, but the ammeter indicates the resistor current ***I*** plus the voltmeter current ***Iv***.

$$R = \frac{E}{I + I_V}$$

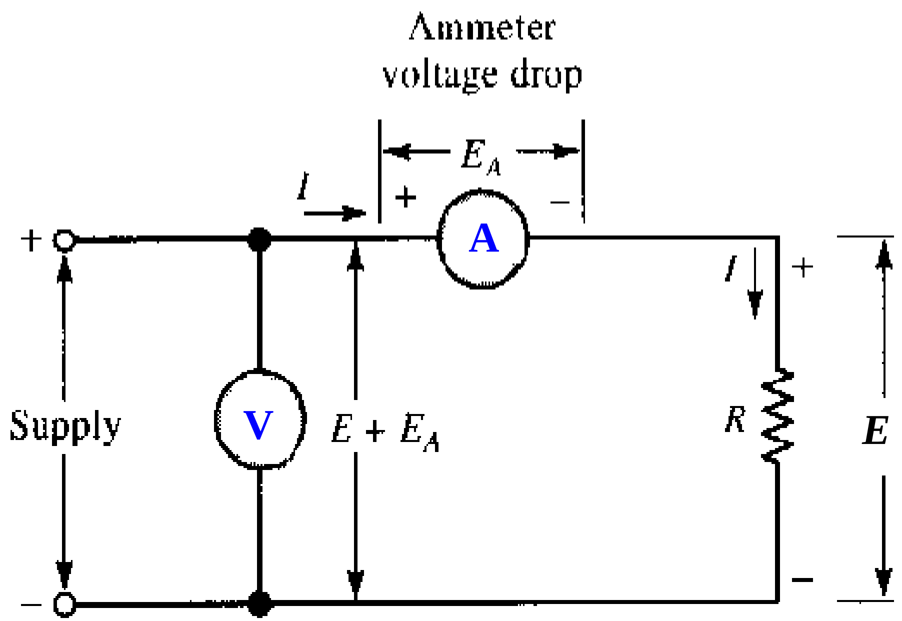


(a) Voltmeter connected across load

RESISTANCE MEASUREMENTS

In this configuration, the ammeter is directly in series with resistor *R*. In this case the voltmeter indicates the resistor voltage *E* plus the ammeter voltage drop E_A.

$$R = \frac{E + E_A}{I}$$



(b) Voltmeter connected across supply

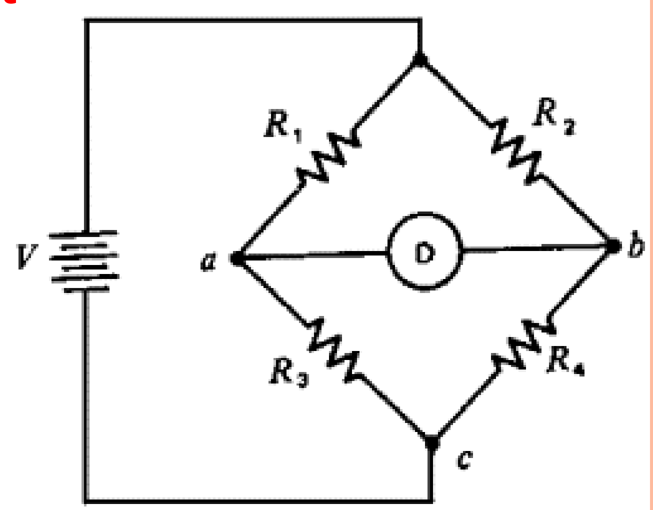
Bridge Circuits

Bridge circuits, which are instruments for making *comparative measurements*, are widely used to measure resistance, inductance, and capacitance

Bridge circuits operate on a *null-indication* principle, which means the measurement is independent of calibration, this makes the measurement device *very accurate*.

Bridge circuits are also frequently used in control circuits.

When used in such applications, one arm of the bridge should contains a resistive element that is sensitive to the physical parameter (Such as: temperature, pressure, etc.) being controlled.



Bridge Circuits: Advantages & Disadvantages

Advantages of the bridges:

- Most accurate types of measurement devices.
- Reliable and very easy to perform.
- Small in size and low power operation.
- Less maintenance required.

Disadvantages of the bridges:

- To ensure the highest accuracy of the bridge circuit, the detector need to be extremely sensitive.

Types of Bridge Circuits

DC-Bridges

- Wheatstone Bridge (Varying Resistance to null bridge)
- Current Balance Bridge (Varying Current to null bridge)
- Kelvin Bridge (Measurement of Low Resistance)

AC-Bridges

- Capacitive and inductive bridges.
- Maxwell & Wien Bridge
- Hay Bridge
- Anderson Bridge
- Radio Frequency Bridge
- Schering Bridge

DC BRIDGES – **WHEATSTONE** BRIDGE

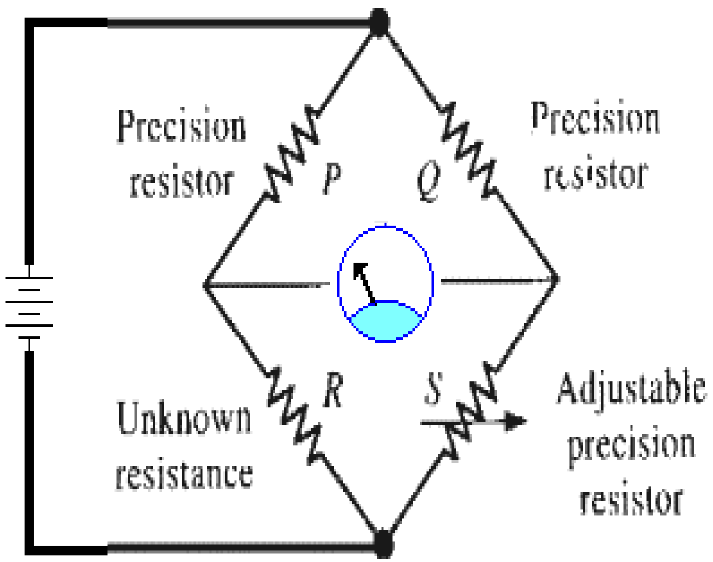
Common name for a normal *DC* bridge is (***Wheatstone Bridge***).

- **Introduced by Sir Charles Wheatstone as early as 1847.**

- **The Wheatstone bridge consists of two parallel resistance branches each branch containing two series elements, usually resistors.**

- **A dc voltage source is connected across this resistance network to provide a source of current through the resistance network.**

- **A *null-detector,* usually a *galvanometer*, is connected between the parallel branches to detect a condition of *balance*.**

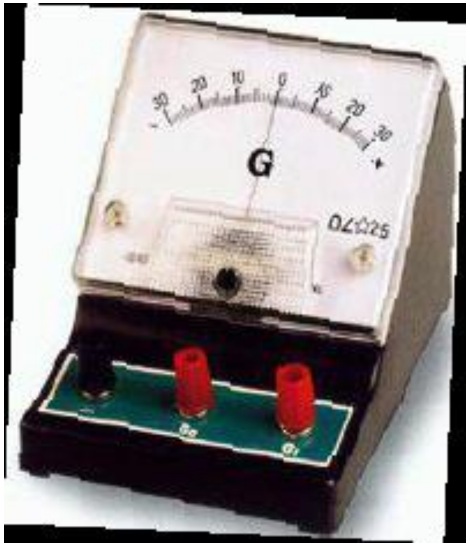


✓ The Wheatstone bridge has been in use longer than almost any other electrical measuring instrument.

✓It is still an accurate and reliable instrument and is heavily used in industry. **Accuracy of 0.1% is quite common with Wheatstone** as **opposed to 3% to 5% error with the ordinary ohmmeters** for resistance measurement.

DC BRIDGES - **WHEATSTONE** BRIDGE

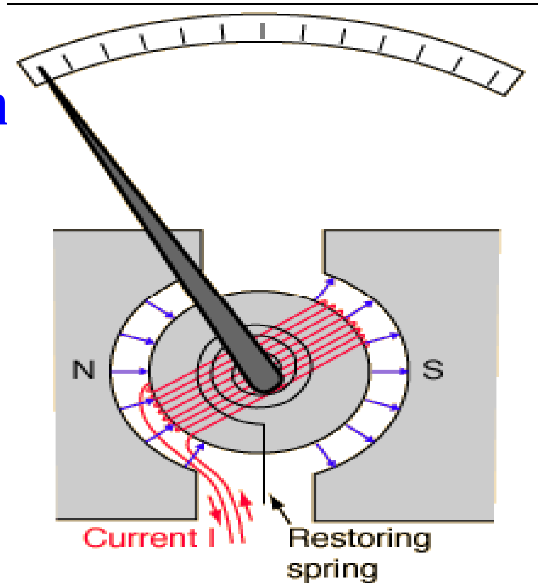
What is a Galvanometer?



Galvanometer is the historical name given to a **moving coil electric** current detector.

When a current is passed through a coil in a magnetic field, the coil experiences a torque proportional to the current.

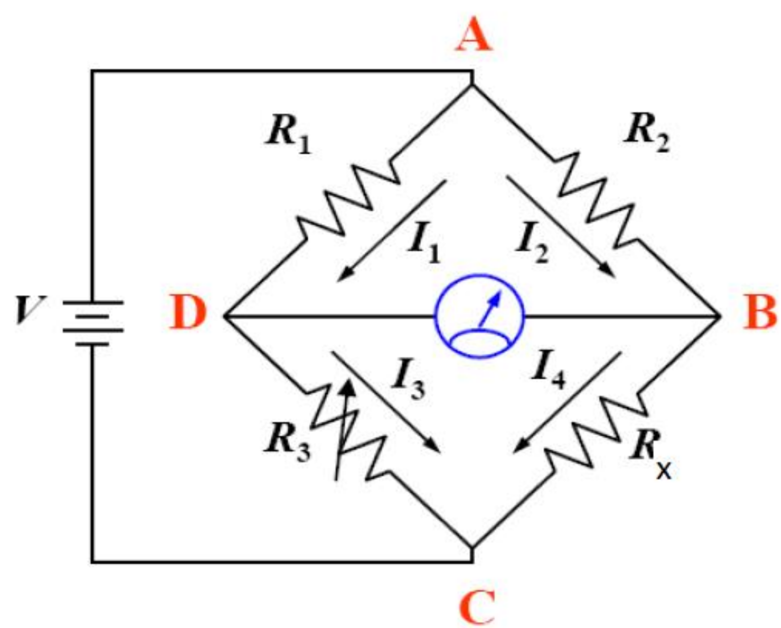
If the coil's movement is opposed by a coil spring, then the amount of deflection of a needle attached to the coil may be **proportional to the current** passing through the coil.



THE WHEATSTONE BRIDGE

- It is the traditional instrument used for making **accurate** measurements of impedance (resistance, reactance, or both)
- Suitable for moderate resistance values: **1 Ω to 10 MΩ**
- Balance condition, when no potential difference across the galvanometer **($V_{DB}=0$)**

null-indication



Ratio (R_2/R_1) called multiplier

- Under this condition

$$V_{AD} = V_{AB}, \qquad I_1R_1 = I_2R_2$$

$$V_{DC} = V_{BC}, \qquad I_3 R_3 = I_4R_x$$

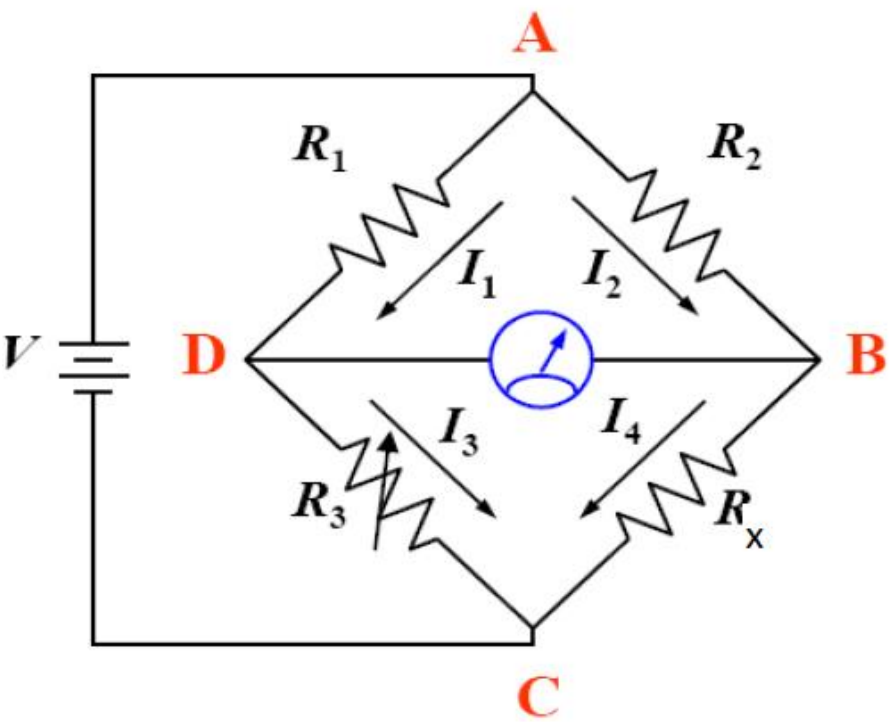
- When current in galvanometer is 0, so $I_1=I_3$ & $I_2=I_4$

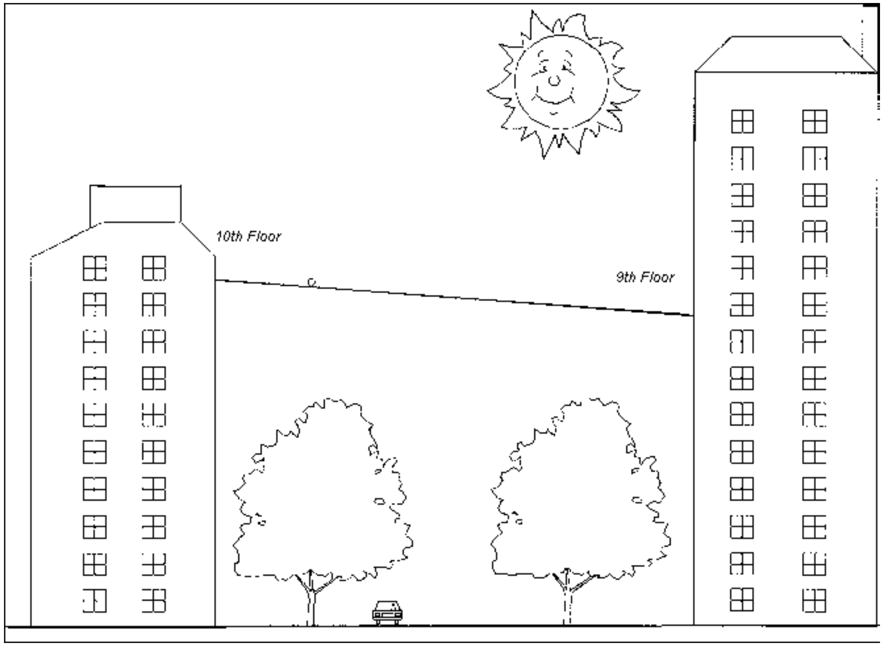
$$\frac{R_1}{R_3} = \frac{R_2}{R_x} \qquad \boxed{R_x = R_3\frac{R_2}{R_1}}$$

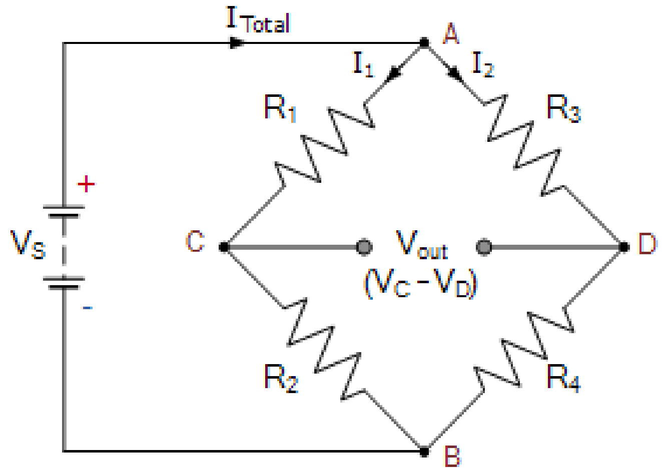
DC BRIDGES - **WHEATSTONE** BRIDGE

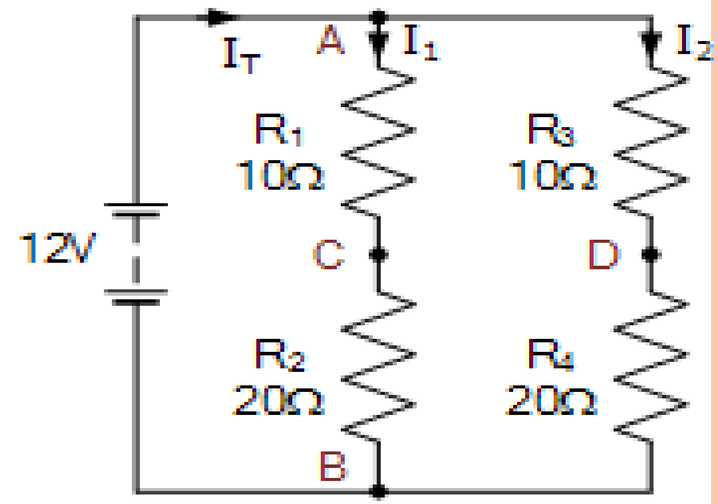
Voltage Potential and Current Movement

The **difference** in potential is crucial for current flow-not the value of the potential to ground of the end points.









$I_1 = V \div R = 12V \div (10\Omega + 20\Omega) = 0.4A$

$V_{R2} = I \times R_2 = 0.4A \times 20\Omega = 8$ volts

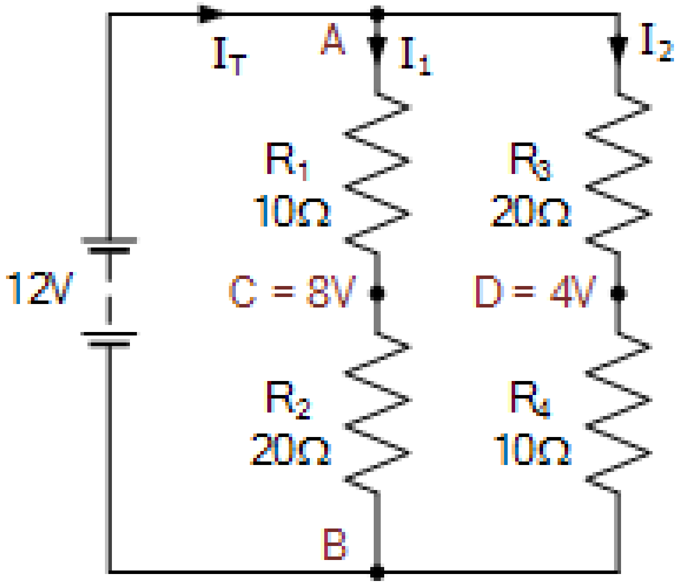
$V_{R1} = 4V$ and $V_{R2} = 8V$

both points have the same value of 8 volts: C = D = 8 volts

the **difference** is: **0 volts**

When this happens, both sides of the parallel network are said to be **balanced** because the voltage at point C is the same value as the voltage at point D.

Consider what would happen when we reverse the position of the two resistors, R_3 and R_4 in the second parallel branch



$V_{R4} = 0.4A \times 10\Omega = 4$ volts

the voltage difference between points C and D will be 4V as: C = 8V and D = 4V

When this happens the parallel network is said to be **unbalanced** as the voltage at point C is at a different value to the voltage at point D.

DC BRIDGES - WHEATSTONE BRIDGE

The bridge consists of an unknown resistance (to be measured) *R*, two precision resistors *P* and *Q*, an adjustable resistor *S*.

To determine *R*, the variable resistance *S* is adjusted until the galvanometer indicates zero voltage.

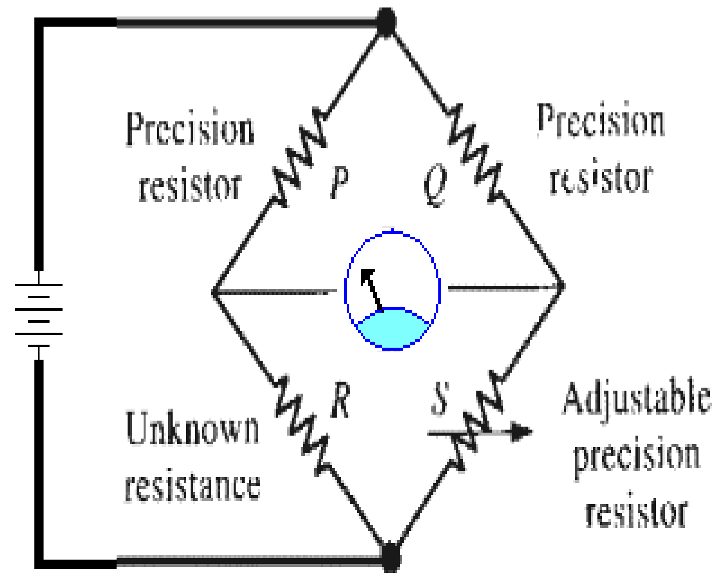
Thus, the bridge is said to be balanced,

When: ***Galvanometer* indicates $\Rightarrow$ 0V,**

$$V_P = V_Q \quad \text{and} \quad V_R = V_S$$

Since $I_G = 0$;

$$I_1 P = I_2 Q \quad \text{and} \quad I_1 R = I_2 S$$

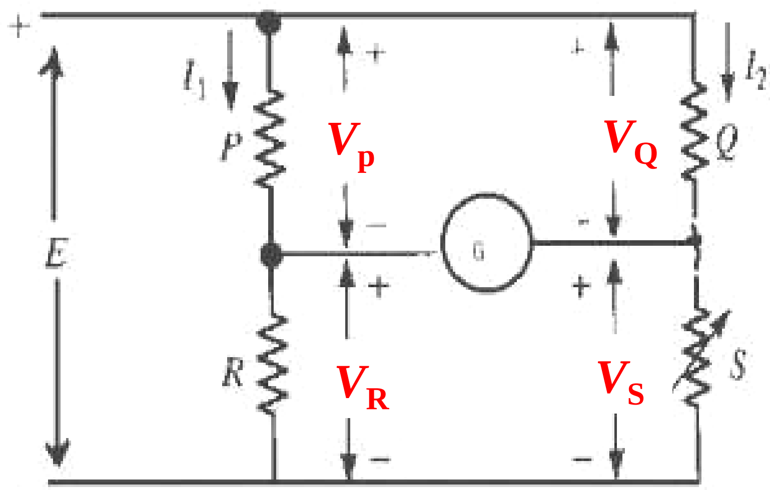


Dividing the 2 equations :

$$\frac{I_1 R}{I_1 P} = \frac{I_2 S}{I_2 Q}$$

Therefore, the unknown resistance (R) :

$$R = \frac{S.P}{Q}$$



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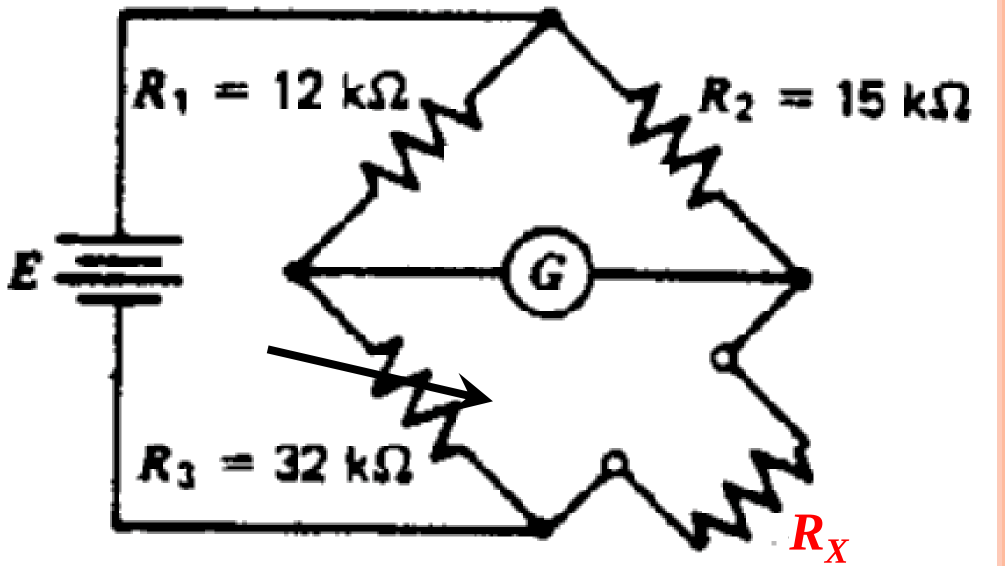
DC Bridges - Wheatstone Bridge

Example 1: **Determine the value of the unknown resistor R_X. Assuming the circuit is balanced at $R_1 = 12k\Omega$, $R_2 = 15k\Omega$ and $R_3 = 32k\Omega$.**



Answer :

$$R_X = \frac{R_2 . R_3}{R_1} = \frac{15 \times 32}{12} = 40k\Omega$$

DC Bridges - **Wheatstone** Bridge

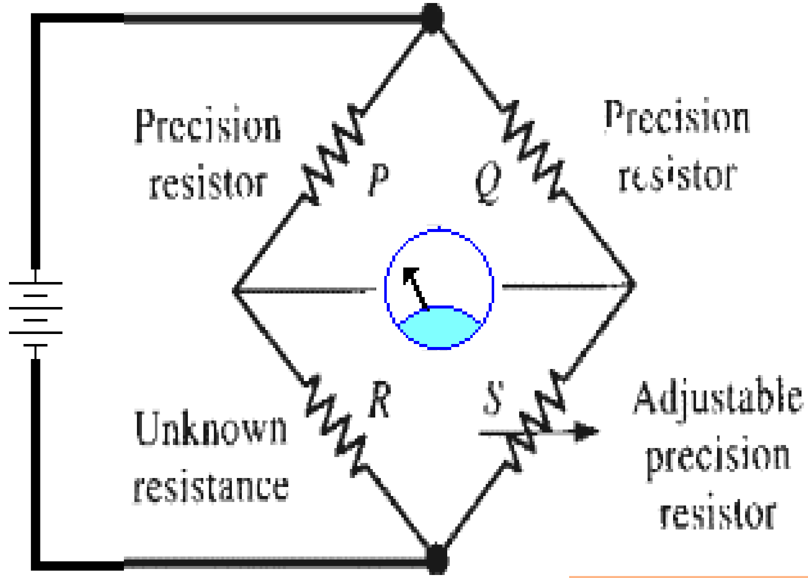
Exercise 1

A Wheatstone bridge has P =5.5kΩ, Q =7kΩ and galvanometer G = 0V when S =5.1kΩ.

a) Calculate the value of R

b) Determine the resistance measurement range for the bridge if S is adjustable from **1kΩ** to **8kΩ**.

$$R = \frac{S.P}{Q}$$



DC Bridges - Wheatstone Bridge

Solution Exercise 1

a) P =5.5kΩ, Q =7kΩ , G = 0V when S =5.1kΩ.

$$R = \frac{S.P}{Q}$$

b)

S =**1kΩ**

S =**8kΩ**

MEASUREMENT ERRORS
IN THE WHEATSTONE BRIDGE

The Wheatstone bridge is mainly used for low value resistance measurements.

1. Error in the Wheatstone bridge is found due to **limiting** errors of the resistors.
2. Insufficient **sensitivity** of null detector.
3. **Change of resistance** due to heating effect.
4. **Thermal emf** in the Bridge or Galvanometer
5. Error due to the **resistance of leads** and circuit contacts.

Parameter of the **Wheatstone** Bridge

1) **Sensitivity of the Wheatstone Bridge**
2) **Voltage Offset**
3) **Offset Current**

PARAMETER OF THE **WHEATSTONE** BRIDGE

1) **Sensitivity of the Wheatstone Bridge**

- **When the bridge is in an unbalanced condition, current flows through the galvanometer, causing a deflection of its pointer.**
- **The amount of deflection is a function of the sensitivity of the galvanometer.**
- **The sensitivity is deflection per unit current.**

$$\boxed{S = \frac{milimeters}{\mu A} = \frac{degrees}{\mu A} = \frac{radian}{\mu A}}$$

- **The *more sensitive* the galvanometer will *deflect more* with the same amount of current**

Therefore, it follows that total deflection D is

$$\boxed{D = SI}$$

where S is as defined and I is the current in microamperes (μA). We might

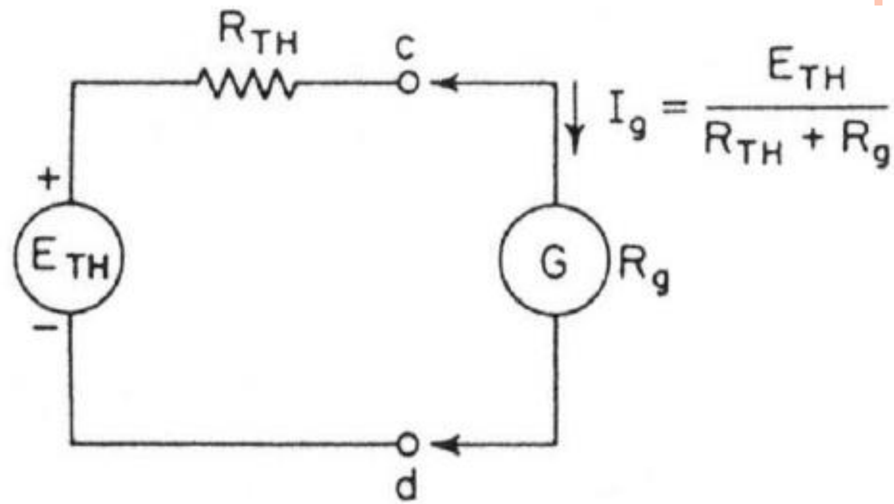
THEVENIN EQUIVALENT CIRCUIT

It is necessary to calculate the galvanometer circuit -- to determine whether or not the galvanometer has the required **sensitivity** to **detect** an **unbalance** conditions.

Different galvanometer not only may require different currents per unit deflection (**current sensitivity**), but also may have a difference **internal resistance**.

☑ **The deflection current** in the galvanometer is,

$$\boxed{I_g = \frac{E_{th}}{R_{th} + R_g}}$$



☑ R_g = the internal resistance in the galvanometer

THEVENIN EQUIVALENT CIRCUIT

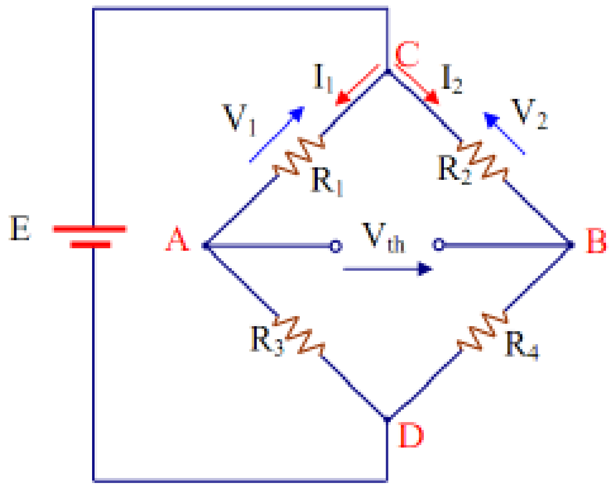
Converting the Wheatstone bridge to its Thevenin equivalent circuit **in order to find the current follows** in the galvanometer:

There are two steps must be taken:

- Finding the **equivalent voltage** when the galvanometer is removed from the circuit (the open voltage between A and B of bridge).
- Finding the **equivalent resistance**, with the battery replaced by its internal resistance (removing the voltage source and makes its side short circuit and removing current source makes its side open circuit)

THEVENIN EQUIVALENT CIRCUIT

Calculate V_{th}

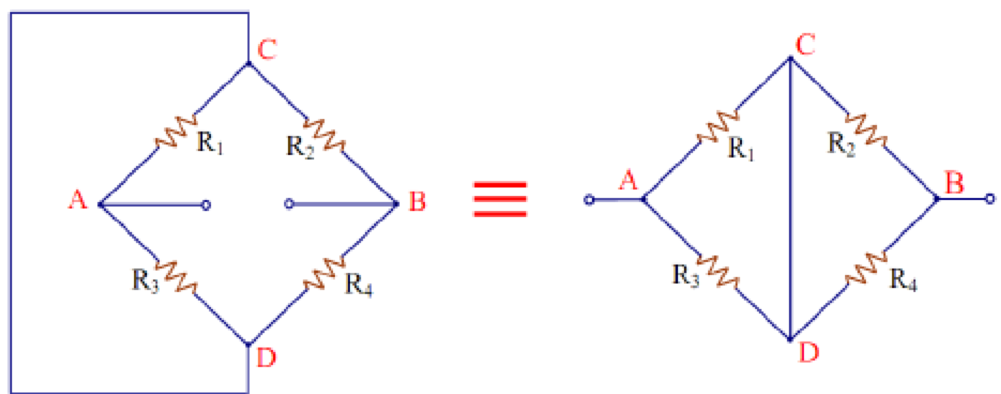


$$V_{th} = V_1 - V_2 = I_1 R_1 - I_2 R_2$$

$$V_{th} = \frac{E}{R_1 + R_3} R_1 - \frac{E}{R_2 + R_4} R_2$$

$$= E \left(\frac{R_1}{R_1 + R_3} - \frac{R_2}{R_2 + R_4} \right)$$

Calculate r_{th}

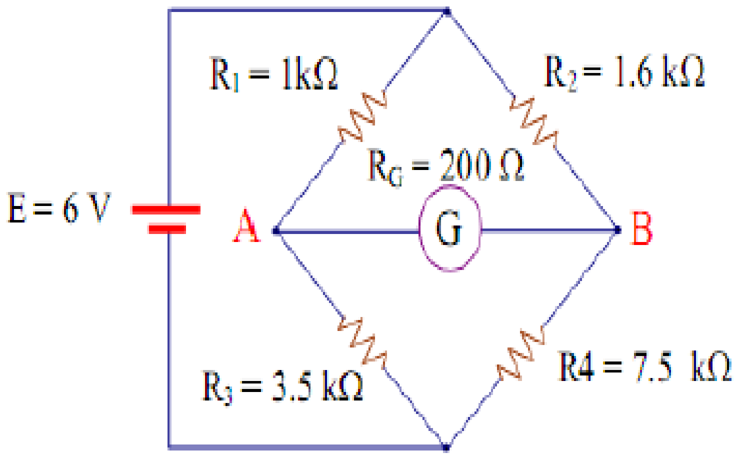


$$R_{Th} = R_{ab} = \left(R_1 // R_3\right) + \left(R_2 // R_4\right)$$

$$r_{th} = \frac{R_1 R_3}{R_1 + R_3} + \frac{R_2 R_4}{R_2 + R_4}$$

Example

Calculate the current passes in the galvanometer of the following circuit.



Solution:

1. Find V_{th}

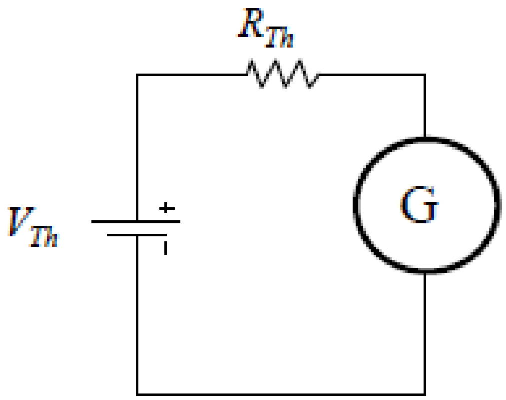
$$V_{th} = E\left(\frac{R_1}{R_1 + R_3} - \frac{R_2}{R_2 + R_4}\right)$$

$$V_{th} = 6 \times \left(\frac{1k\Omega}{1k\Omega + 3.5k\Omega} - \frac{1.6k\Omega}{1.6\,k\Omega + 7.5\,k\Omega}\right) = 0.278V$$

2. Find r_{th}

$$r_{th} = \frac{R_1\,R_3}{R_1 + R_3} + \frac{R_2\,R_4}{R_2 + R_4}$$

$$r_{th} = \frac{1\,k\Omega \times 3.5\,k\Omega}{1\,k\Omega + 3.5\,k\Omega} + \frac{1.6\,k\Omega \times 7.5\,k\Omega}{1.6\,k\Omega + 7.5\,k\Omega} = 2.096\,k\Omega$$



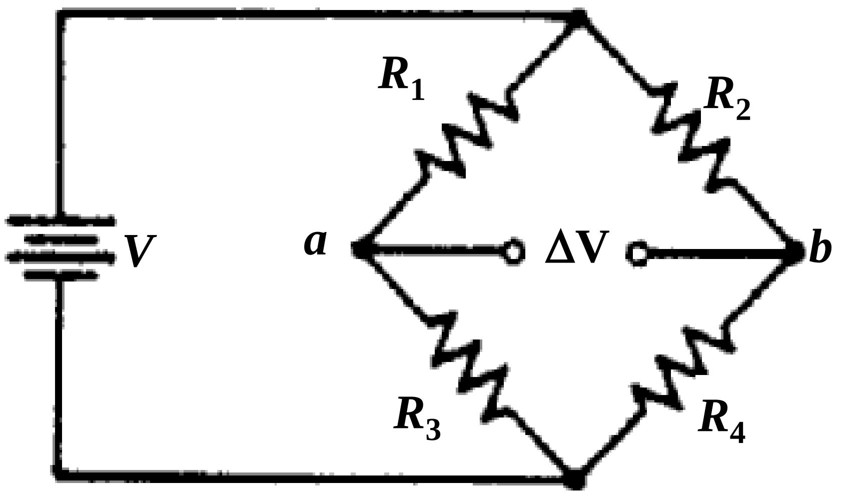
3. Find I_G

$$I_G = \frac{V_{th}}{r_{th} + R_G} = \frac{0.278\ V}{2.096 \times 10^3\ \Omega + 200\ \Omega} = 121.4\,\mu A$$

PARAMETER OF THE **WHEATSTONE** BRIDGE

2) **Voltage Offset**

If we assume the galvanometer impedance is **infinite**, which is an open circuit.



If the bridge is balanced, then:

$$\Delta V = 0 \; and \; V_a = V_b$$

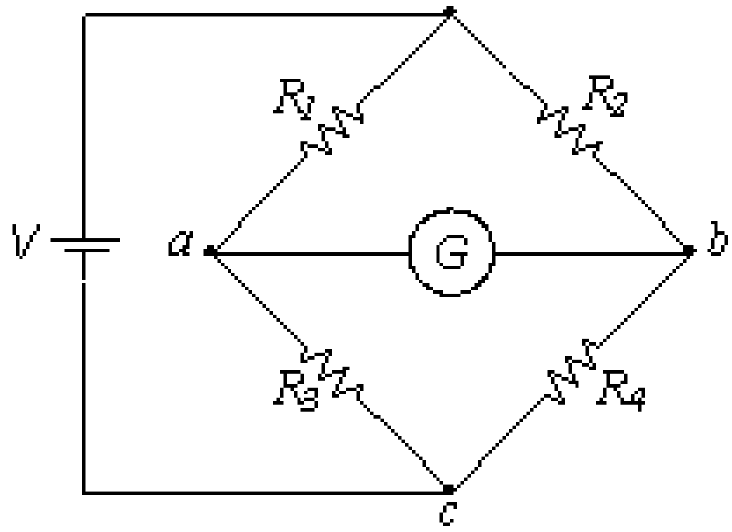
If the bridge is unbalanced , then the potential difference is :

$$\Delta V \neq 0 = V_a - V_b = V \cdot \frac{R_3}{R_1 + R_3} - V \cdot \frac{R_4}{R_2 + R_4} = V \cdot \left[\frac{R_3}{R_1 + R_3} - \frac{R_4}{R_2 + R_4} \right]$$

Parameter of the Wheatstone Bridge

Exercise 2

A bridge circuit in Figure below is used for potential measurement nulls when $R1=R2=1k\Omega$, $R3=605\Omega$, and $R4=500\Omega$ with a **10V** supply. **Find the offset voltage**.



$$\Delta V \neq 0 = V_a - V_b = V \cdot \frac{R_3}{R_1 + R_3} - V \cdot \frac{R_4}{R_2 + R_4} = V \cdot \left[\frac{R_3}{R_1 + R_3} - \frac{R_4}{R_2 + R_4} \right]$$

Parameter of the Wheatstone Bridge

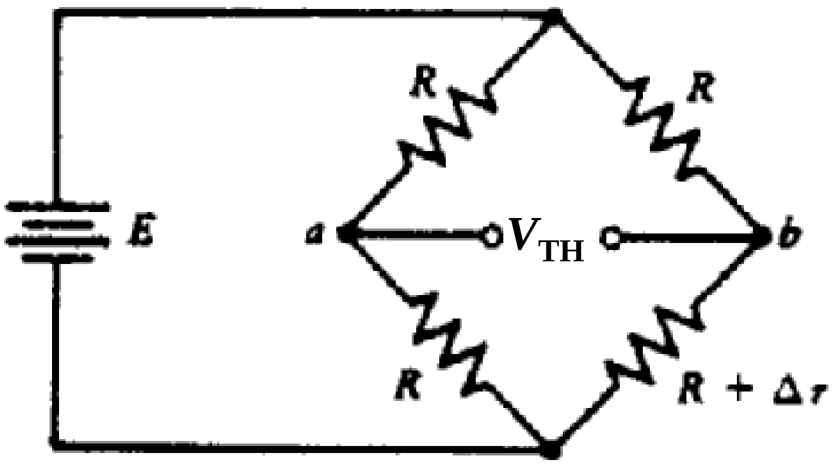
Solution Exercise 2

3) Offset Current

Offset current **is the current drawn by the galvanometer when the bridge is unbalanced**.

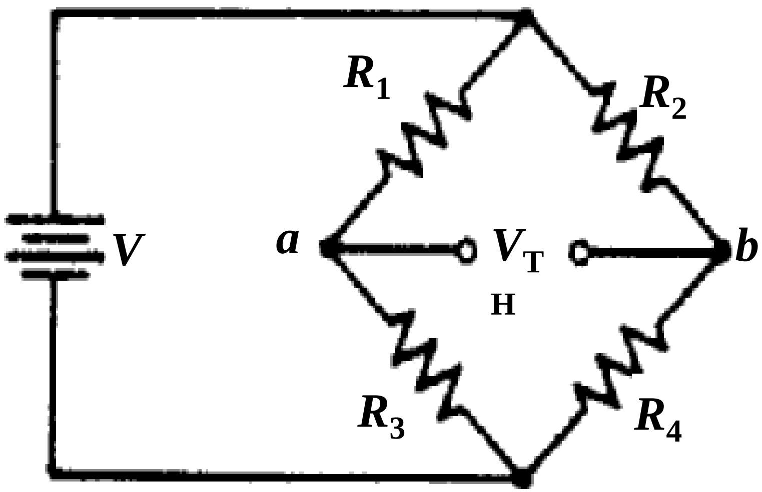
The easiest way to determine this offset current is to find the Thevenin equivalent circuit between point ***a*** and ***b*** of the bridge.

The Thevenin voltage and resistance are the voltage and resistance measured between point ***a*** and ***b***, so



Parameter of the Wheatstone Bridge

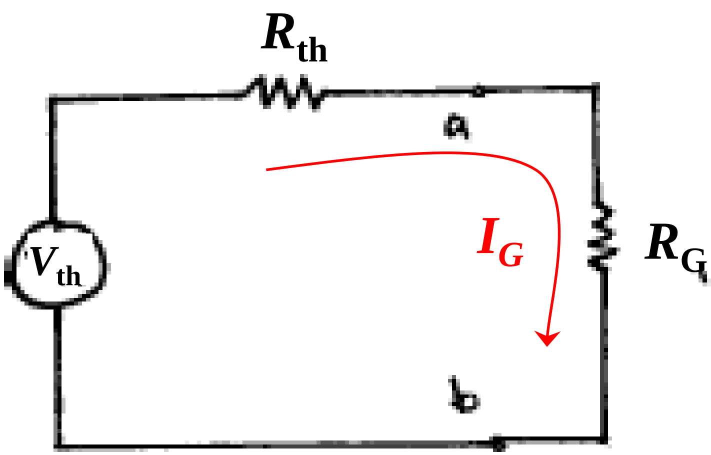
The Thevenin voltage and resistance are calculated for the **unbalanced** bridge as below:



$$V_{Th} = \Delta V = \frac{R_3}{R_1 + R_3} V - \frac{R_4}{R_2 + R_4} V$$

$$R_{th} = R_1 // R_3 + R_2 // R_4$$

And the offset current can be calculated:



Hence, $I_G \neq 0$

$$\boxed{I_G = \frac{V_{th}}{R_G + R_{th}}}$$

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Example 2:

A bridge circuit has resistance of $R_1 = R_2 = R_3 = 2.5k\Omega$ and $R_4 = 2k\Omega$ and a 15V supply. If galvanometer with a 50Ω internal resistance is used for a detector, **find the offset current**.

Solution

$$V_{Th} = \frac{R_2R_3 - R_1R_4}{\left(R_1 + R_3\right)\left(R_2 + R_4\right)}V = \frac{2.5k \times 2.5k - 2.5k \times 2k}{(2.5k + 2.5k)(2.5k + 2k)}15 = 0.833V$$

$$R_{Th} = \frac{R_1R_3}{R_1 + R_3} + \frac{R_2R_4}{R_2 + R_4} = \frac{2.5k \times 2.5k}{2.5k + 2.5k} + \frac{2.5k \times 2k}{2.5k + 2k} = 2.361k\Omega$$

The offset current:

$$I_G = \frac{V_{Th}}{R_G + R_{Th}} = \frac{0.833}{50 + 2.361k} = 3.45 \times 10^{-4} A$$

Balanced Bridge Useful electronics APPLICATIONS

- Used to measure changes in light intensity, pressure or strain.
- The types of **resistive sensors** that can be used within a wheatstone bridge circuit include:

✓ Photoresistive sensors (LDR's)

✓ Positional sensors (potentiometers),

✓ Piezoresistive sensors (strain gauges) and

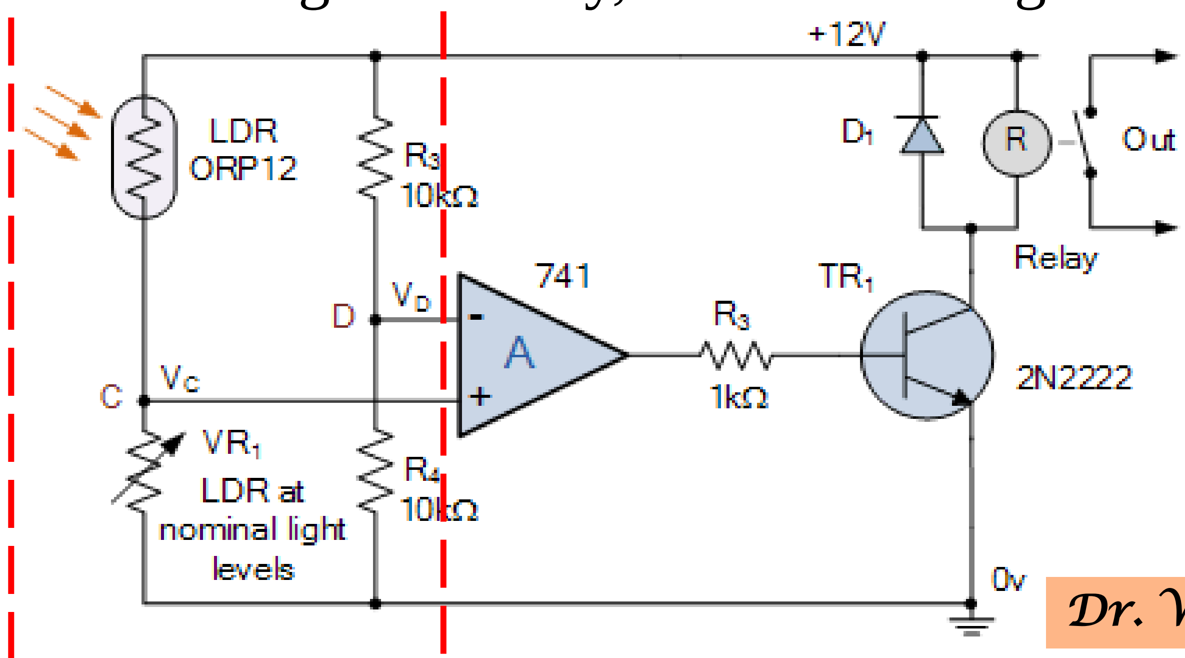
✓ Temperature sensors (thermistor's), etc.

Application: Wheatstone Bridge Light Detector

One of the resistors within the bridge network is replaced by a light dependent resistor, or LDR.

An LDR, also known as a cadmium-sulphide (Cds) photocell, is a passive **resistive sensor** which **converts changes in visible light** levels **into a change in resistance** and hence a voltage.

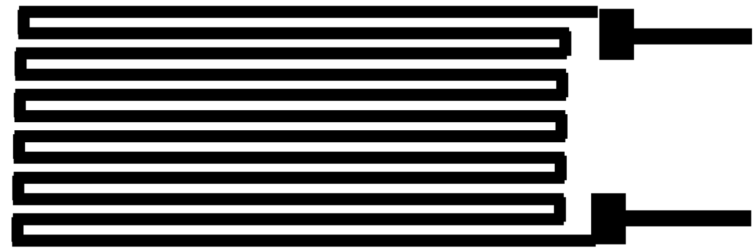
Light dependent resistors can be used for monitoring and measuring the level of light intensity, or whether a light source is ON or OFF.

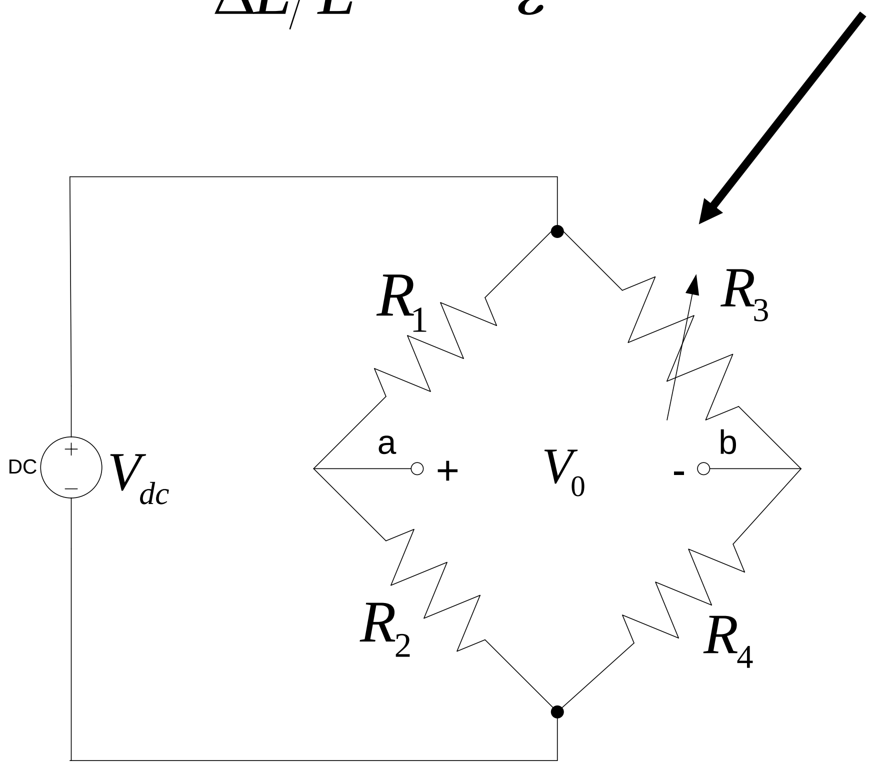


APPLICATION: STRAIN GAUGE

Gauge factor, G_F

$$G_F = \frac{\Delta R / R_0}{\Delta L / L} = \frac{\Delta R / R_0}{\varepsilon}$$



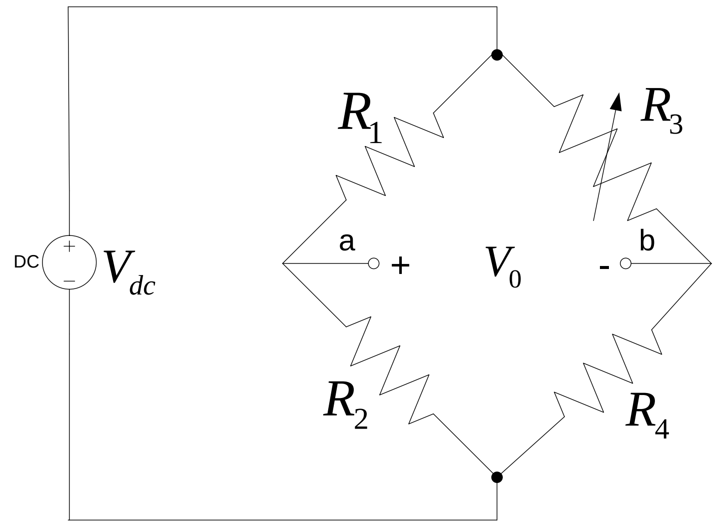


$$R_3 = R_0 + \Delta R = R_0\left(1 + \Delta R / R_0\right)$$

$$R_3 = R_0\left(1 + G_F\,\varepsilon\right)$$

APPLICATION: MEASURING STRAIN

$$V_0 = V_{dc}\left[\frac{R_2}{R_1+R_2} - \frac{R_4}{R_3+R_4}\right]$$



$$V_0 = V_{dc}\left[\frac{R_2}{R_1+R_2} - \frac{R_4}{R_0\left(1+G_F\varepsilon\right)+R_4}\right]$$

$$R_3 = R_0\left(1+G_F\varepsilon\right)$$

Solving for ε

$$\varepsilon = \frac{R_4}{G_F R_0}\left[\frac{1}{\left(\dfrac{R_2}{R_1+R_2} - \dfrac{V_0}{V_{dc}}\right)} - 1\right] - \frac{1}{G_F}$$

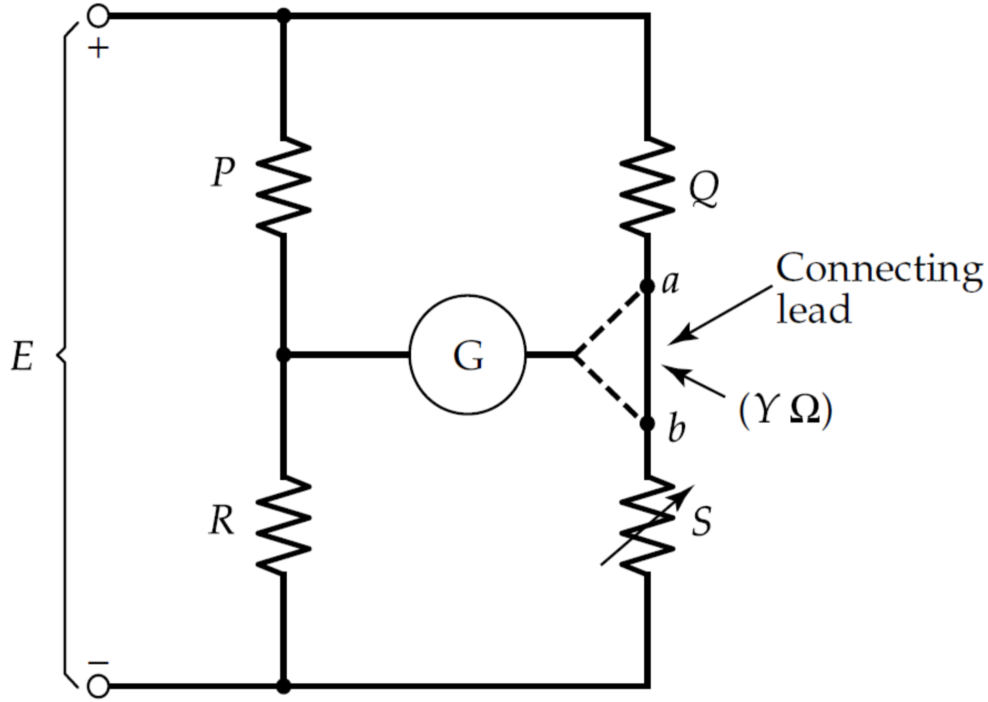
Wheatstone Bridge Limitation

- **It is often necessary to make measurements of low resistances, such as for samples of wires or low values of meter-shunt resistors.**

- **Resistors in the range of approximately $1\mu\Omega$ to 1Ω may be measured with a high degree of accuracy using a bridge called the *Kelvin bridge*.**

- **Kelvin bridge is a modified version of the Wheatstone bridge. The purpose of the modification is to eliminate the effects of contact, and lead resistance when measuring unknown low resistances.**

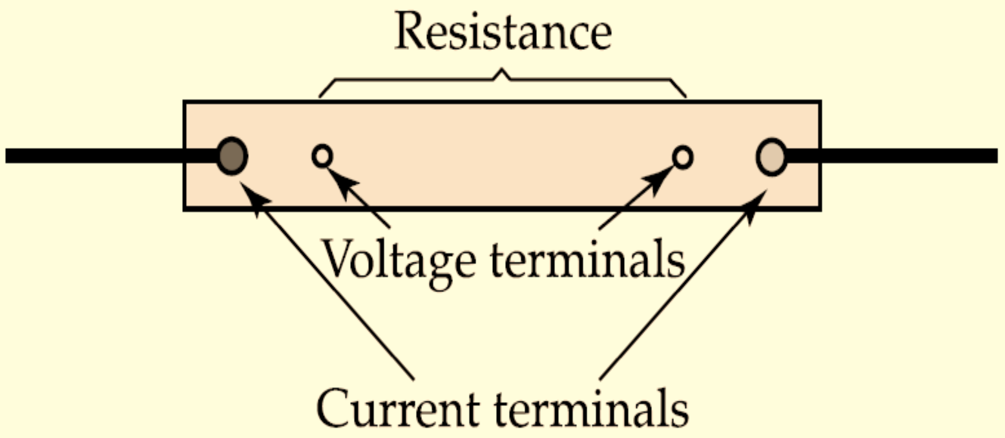
Range of Measurement



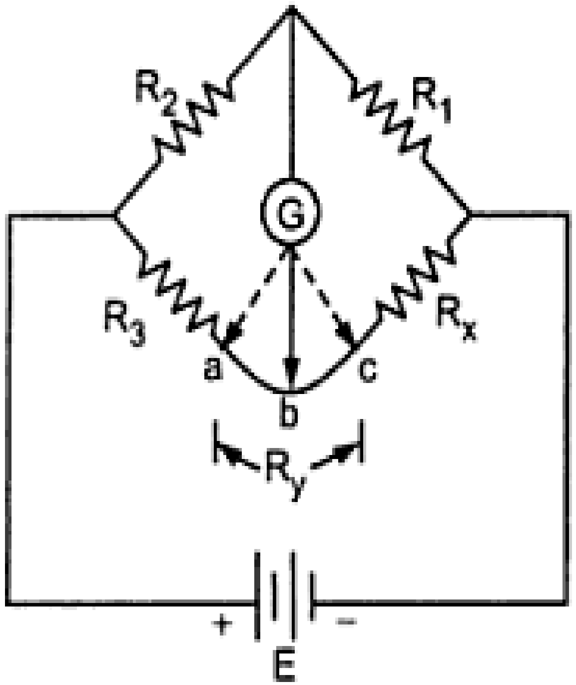
The connecting leads in a Wheatstone bridge circuit can introduce errors when measuring very low resistances. The *Y* resistance could be taken as part of *Q* or part of *S*.

Four Terminal Resistors

Four-terminal resistors have current terminals and voltage terminals. The resistance is defined as that between the voltage terminals, so that there is no error introduced by contact voltage drops at the current terminals.



KELVIN'S BRIDGE



Kelvin bridge

In the Wheatstone bridge, the bridge contact and lead resistance causes significant error, while measuring low resistances. Thus for measuring the values of resistance below 1Ω, the modified form of Wheatstone bridge is used, known as Kelvin bridge. The consideration of the effect of contact and lead resistances is the basic aim of the Kelvin bridge.

The Fig. is the basic circuit of the Kelvin bridge.

Dr. Wael Salah

Kelvin's Bridge

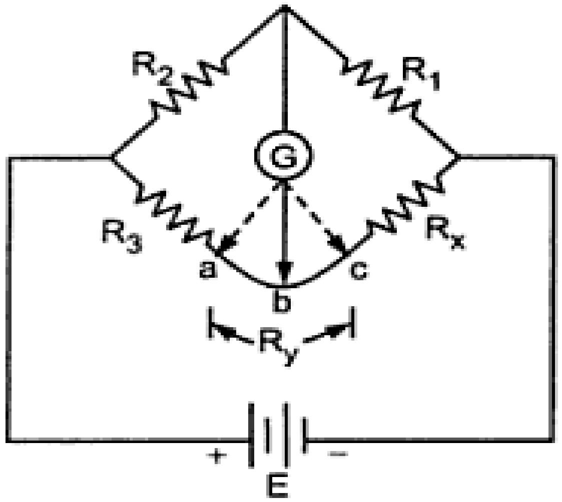
The resistance R_y represents the resistance of the connecting leads from R_3 to R_x. The resistance R_x is the unknown resistance to be measured.

The galvanometer can be connected to either terminal a, b or terminal c. When it is connected to a, the lead resistance R_y gets added to R_x hence the value measured by the bridge, indicates much higher value of R_x.

If the galvanometer is connected to terminal c, then R_y gets added to R_3. This results in the measurement of R_x much lower than the actual value.

The point b is in between the points a and c, in such a way that the ratio of the resistance from c to b and that from a to b is equal to the ratio of R_1 and R_2.

$$\therefore \qquad \frac{R_{cb}}{R_{ab}} = \frac{R_1}{R_2}$$



Now the bridge balance equation in its standard form is,

$$R_1R_3 = R_2R_x$$

KELVIN'S BRIDGE

But R_3 and R_x now are changed to $R_3 + R_{ab}$ and $R_x + R_{cb}$ respectively due to lead resistance.

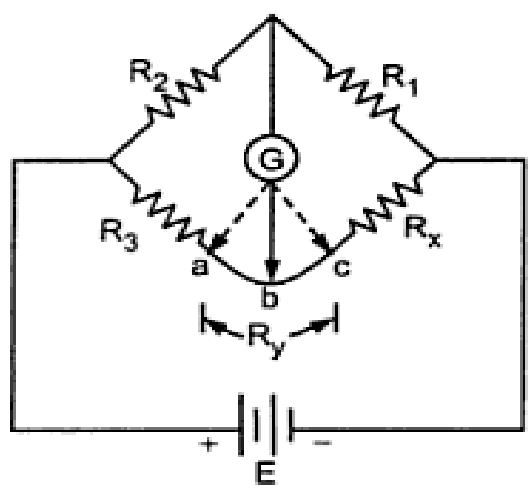
$$\therefore \quad R_1(R_3 + R_{ab}) = R_2(R_x + R_{cb}) \quad \ldots (3)$$

$$\therefore \quad (R_x + R_{cb}) = \frac{R_1}{R_2}(R_3 + R_{ab}) \quad \ldots (4)$$

Now we have, $\dfrac{R_{cb}}{R_{ab}} = \dfrac{R_1}{R_2}$

$$\therefore \quad \frac{R_{cb}}{R_{ab}} + 1 = \frac{R_1}{R_2} + 1 \quad \ldots \text{adding 1 to both sides}$$

$$\frac{R_{cb} + R_{ab}}{R_{ab}} = \frac{R_1 + R_2}{R_2} \quad \ldots (5)$$



sistance

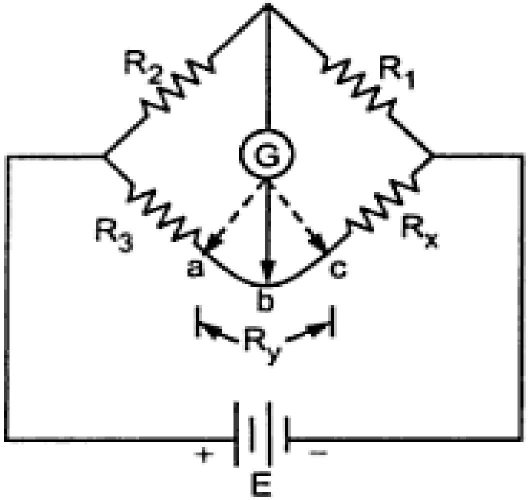
But $R_{cb} + R_{ab} = R_y$

Substituting in (5) we get,

$$\frac{R_y}{R_{ab}} = \frac{R_1 + R_2}{R_2} \quad \ldots (6)$$

$$\therefore \quad R_{ab} = \frac{R_2 R_y}{R_1 + R_2} \quad \ldots (7)$$

KELVIN'S BRIDGE



Now $$R_{cb} + R_{ab} = R_y$$

$\therefore$ $$R_{cb} = R_y - R_{ab} \qquad \dots (8)$$

Substituting (7) into (8),

$$R_{cb} = R_y - \frac{R_2 R_y}{R_1 + R_2}$$

$$= R_y \left[1 - \frac{R_2}{R_1 + R_2}\right]$$

$\therefore$ $$R_{cb} = \frac{R_1 R_y}{R_1 + R_2} \qquad \dots (9)$$

Substituting these values of R_{cb} and R_{ab} in the equation (4) we get,

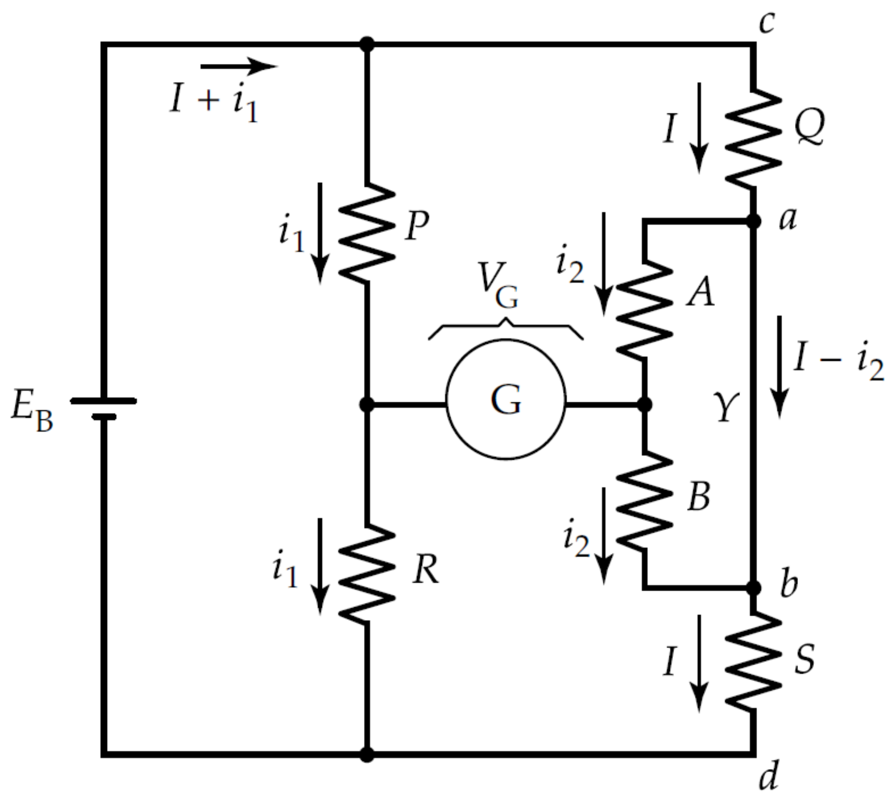
$$R_x + \frac{R_1 R_y}{R_1 + R_2} = \frac{R_1}{R_2}\left(R_3 + \frac{R_2 R_y}{R_1 + R_2}\right)$$

$$\therefore R_x + \frac{R_1 R_y}{R_1 + R_2} = \frac{R_1 R_3}{R_2} + \frac{R_1 R_y}{R_1 + R_2}$$

$\therefore$ $$\boxed{R_x = \frac{R_1 R_3}{R_2}} \qquad \dots (10)$$

This concludes that the effect of the resistance of the connecting lead a to c has been eliminated by connecting the galvanometer at the intermediate point c

Kelvin's Double Bridge



➔ The Kelvin bridge is essentially a Wheatstone bridge with two additional resistors (*A* and *B*). These are included to eliminate the voltage drop across *Y* from the balance equation.

Kelvin's Double Bridge

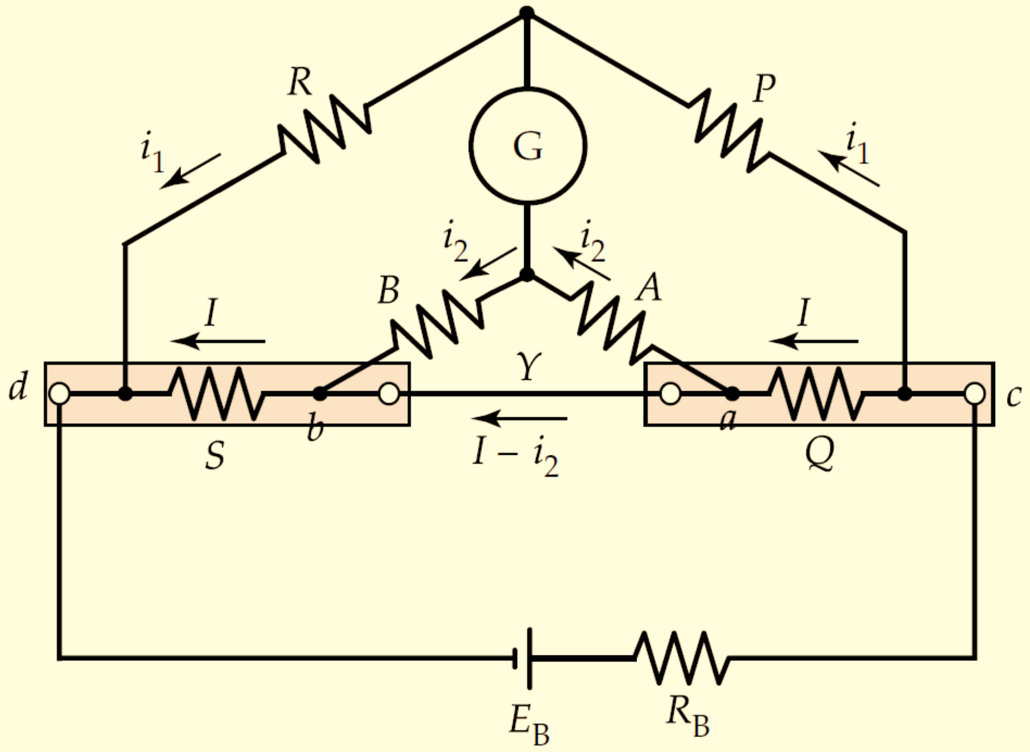


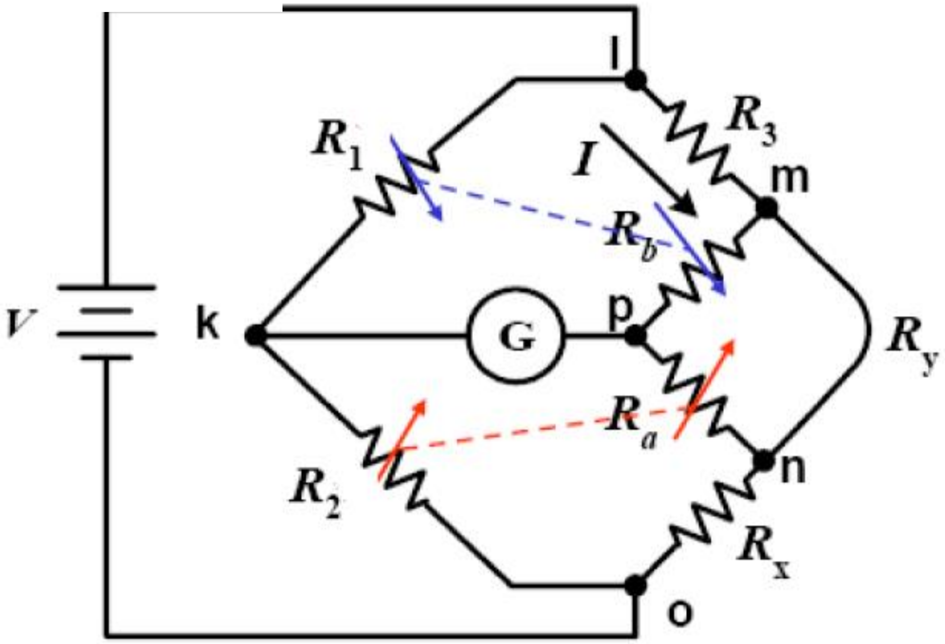
Figure 8-8 Kelvin bridge for very low resistance measurement. *S* is a standard four-terminal resistor, *Q* is the resistor to be measured, and *P*, *R*, *A*, and *B* are precision resistors. When the bridge is balanced, $Q = S\,P/R$.

DC BRIDGES – KELVIN DOUBLE BRIDGE

- It is used to solve the problem of connecting leads
- it has two balanced ratio
- can measure small resistance (**0.0001Ω**) with error 0.1%

The balance conditions are :

- $V_{lk} = V_{lmp}$
- $V_{ok} = V_{onp}$



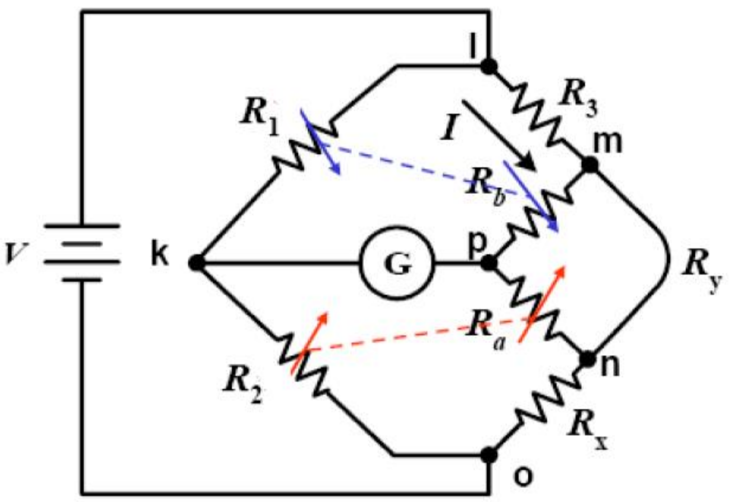
$$V_{lK} = V\frac{R_1}{R_2 + R_1} \qquad (1)$$

$$V_{lo} = I[R_3 + R_x + (R_b + R_a)//R_y]$$

$$I_{Ry}R_y = I_{mpn}(R_b + R_a)$$

$$V_{lmp} = IR_3 + I * \frac{R_y}{R_a + R_b + R_y}R_b \qquad (2)$$

From Eqs.(1)-(2) and rearrange, we have



$$R_X = R_3\frac{R_2}{R_1} + \frac{R_bR_y}{R_a + R_b + R_y}\left(\frac{R_2}{R_1} - \frac{R_a}{R_b}\right)$$

With the balance condition $\frac{R_a}{R_b} = \frac{R_2}{R_1}$

$$R_x = R_3\frac{R_2}{R_1}$$