Chapter 1 (Text Book)

Measurement and Error

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Objectives:

- To introduce the elements of measurement systems
- To introduce the functions and characteristics of instruments
- To define measurement errors
- To know the key terms of measurement errors
- To introduce the different types of errors
- To calculate errors in measurements

Measurement System

- A measurement system is a system that converts an unknown quantity being measured to a numerical unit using an instrument.

- Measurement:

The use of an instrument or device as a physical mean to find a **value** or **quantity**.

- **Result**: Number + measured unit.

  E.g.: **6.8** kg / ms^2

Terms

- **Measurand** – the unknown quantity to be measured.
- **Instrument** – physical device used to determine the measurand numerically.

Accuracy

- **Closeness** with which an instrument reading approaches the **true value** of the quantity measured.

- Example:
  Reading from instrument A, $\ell = 3.82$cm
  Reading from instrument B, $\ell = 3.91$ cm
  True value, $\ell = 3.90$cm

- Conclusion: Instrument B is **more accurate**.

Precision

It is a measure of **reproducibility** of the measurement

- E.g. given a fixed value of quantity, precision is a measure of the **degree of agreement** within a group of measurements.
- It is composed of 2 characteristics:

a) Conformity

b) Number of significant figures

Instrument A, I= 3.82, 3.82, 3.81, 3.82…

Instrument B, I = 3.82, 3.84, 3.83, 3.80…

Conclusion: Instrument A is **more precise.**

Accuracy and Precision

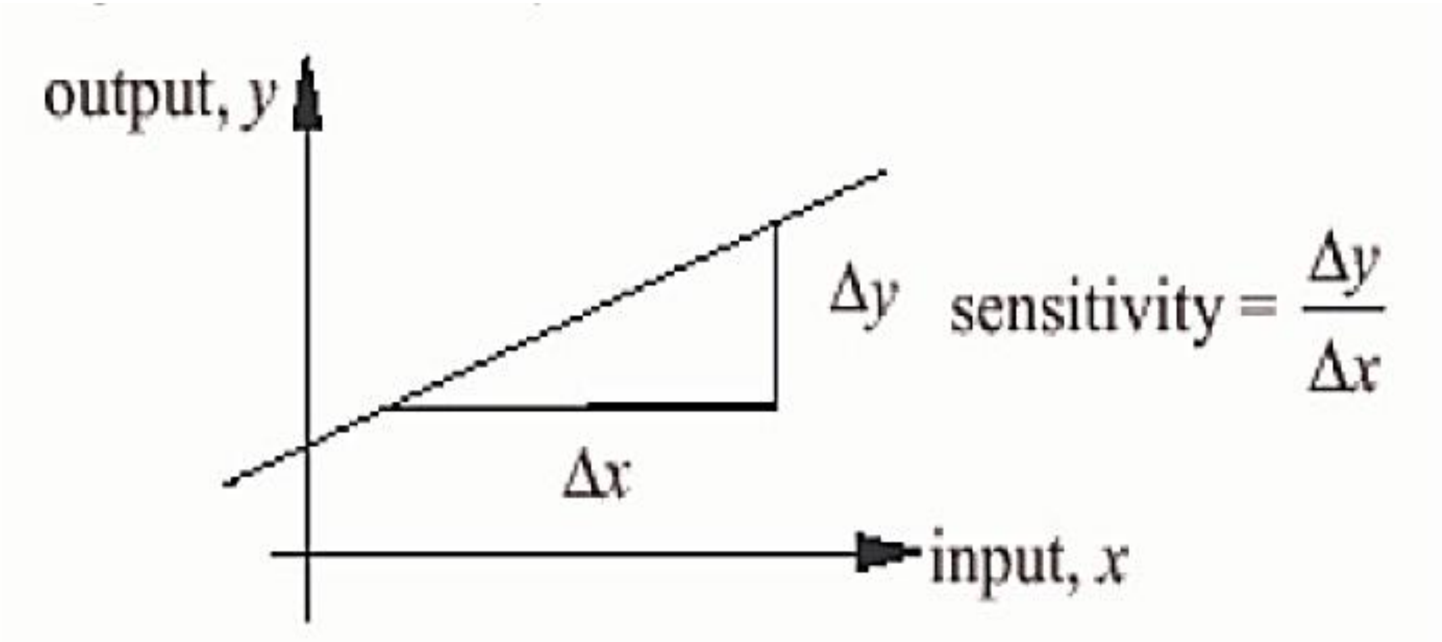
- True value, l = 1.50mm
  Instrument A, l = 1.475mm
  Instrument B, l = 1.49mm

- Conclusion:
  Instrument A is more precise
  Instrument B is more accurate

Precision and accuracy of measurement.

Sensitivity

- The **ratio** of the magnitude of the output signal or **response** to a change the magnitude of input signal.



Example: Sensitivity

- A wheastone bridge requires a change of 7Ω in an unknown arm of the bridge to produce a change in deflection of 3mm of the galvanometer. Determine the sensitivity.

- **Sensitivity = $\frac{\text{magnitude of output response}}{\text{magnitude of input}}$**

$$= \frac{3mm}{7\Omega} = \underline{0.429 mm/\Omega}$$

Example: Sensitivity

The following resistance values of a platinum resistance thermometer were measured at a range of temperatures. Determine the measurement sensitivity of the instrument in ohms/°C.

Resistance (Ω)	*Temperature* (°C)
307	200
314	230
321	260
328	290

Solution

If these values are plotted on a graph, the straight-line relationship between resistance change and temperature change is obvious.

For a change in temperature of 30°C, the change in resistance is 7 Ω. Hence the measurement sensitivity $= 7/30 = 0.233\ \Omega/°C$.

Terms

- Accuracy
- Precision
- Sensitivity

- Reliability
- Resolution
- Response time
- Frequency response
- Switching time
- Bandwidth
- Hysteresis

- True value
- Measured value
- Nominal value
- Absolute & static error
- Relative absolute error

Reliability

The period an instrument can maintain its accuracy and precision.

- Example:

  After two years of using an instrument…

  Accuracy of Instrument A drops 1%

  Accuracy of Instrument B drops 5%

- Conclusion: Instrument A is **more reliable.**

Resolution/ Discrimination

The **smallest** increment in **input** which can be detected with certainty by an instrument.

- Example:

  A mercury thermometer reacts after every 0.5°C change of ambient temperature.

- This thermometer won't have any reaction if the change in temperature is 0.4°C or less. And it will move a step if the change is 0.6°C

Response time

The time taken for an instrument to **sense** an input and give a **steady state** output.

- Example:

  A mercury thermometer reacts for every 0.5°C change of ambient temperature which requires 1.5s to settle.
- If the temperature changes rapidly every 1s, then this thermometer would never give a proper value.

Bandwidth

The range of frequency that an equipment can sense and give **satisfactory** output response.

Example:

- The bandwidth of our ears is from 20Hz to 20kHz. Any sound that is outside this range is undetectable.
- Oscilloscope can display signals with frequency range 0-20kHz. Freq. > 20kHz can not be displayed.

Error Calculation

True value

- The actual value of a quantity.
- It is almost impossible to obtain in practice.
- For example:

  Light speed = 299792458.63… m/s

Measured value

- Value of a measurand as indicated by an instrument.

- It should always be followed by its **uncertainty** in measurement.

- For example:

$\ell = (3.5 \pm 0.1)$ cm

$R = (102.5 \pm 0.2)\ \Omega$

Nominal value

Value of the quantity specified by the **manufacturer**

- It is followed by tolerance level

- For example:

$\ell = 3.5\text{cm} \pm 10\%$

$R = 10\text{k}\,\Omega \pm 5\%$

(True value is between $9.5\text{k}\,\Omega$ and $10.5\text{k}\,\Omega$)

Absolute Error

- Absolute Error: The difference between the measured value and the true value of the quantity

$$\delta A = A_m - A_t$$

δA = Absolute error

A_m = measured value,

A_t = true value

Relative Error

- Relative error:
- The **ratio** of absolute error to the true value

➔Relative error (**as a fraction**):

$$\varepsilon_r = \delta A / A_t \ = A_m - A_t / A_t$$

➔Relative Percentage Error (**as Percentage %**)

$$\% \text{ Error } = \frac{AbsuluteError}{AcualValue} \times 100\%$$

Guarantee/Limiting Error

Guarantee/Limiting Error

- Manufacturer must always guarantee a certain accuracy of their product **numerically** to ensure the quality of their instrument.
- The manufacturer specifies the **deviations from the nominal value** of a particular quantity – defined as limiting errors or guarantee errors.

Actual Value

- Actual value $A_a = A_n \pm \delta A$

  A_a = Actual value

  A_n = Nominal value

  δA = limiting error

- Hence, A_a satisfies:

  $A_n - \delta A \leq \mathbf{A_a} \leq A_n + \delta A$

- Example: $R = (102.5 \pm 0.2)\ \Omega$

  Magnitude of R is $102.3\ \Omega \leq R \leq 102.7\ \Omega$

Relative limiting error

- Relative limiting error = $\varepsilon_r = \delta A/A_n$

  % Relative error = ε_r X 100%

- Example:

  C = 10μF ± 5%

  $\delta A = \varepsilon_r A_n = 0.05 \times 10\mu F = 0.5uF$

  $9.50\mu F \leq A_a \leq 10.5\mu F$

Example:

A 0–150V voltmeter has a guarantee accuracy of 1% at full-scale reading. The voltage measured by this instrument is 83V.

a. Calculate the limiting error in per cent.
b. Calculate Actual value.

Solution:

a. L.E. = 0.01x150V = 1.5V

The %E at a meter indication of 83V

%E = (1.5/83)*100% =1.81%

b. Actual value = 83 ± 1.81% = 83V ± 1.5V

Combination of limiting errors

- If the errors in the component quantities, dx_i are represented by $\pm\delta x_1, \pm\delta x_2, \ldots, \pm\delta x_n$,
- the limiting error δy in y is given by :

$$\boldsymbol{\delta y = \pm (\delta x_1 + \delta x_2 + \ldots \delta x_n)}$$

➔ The limiting error obtained for the **worst** possible combination by <u>direct sum</u> of all possible Errors.

$$X = (x_{1} * x_2)/x_3$$

The resultant Error = $\boldsymbol{\delta y = \pm (\delta x_1 + \delta x_2 + \delta x_3)}$

Total error when combining multiple measurements

Example:

A rectangular-sided block has edges of lengths a, b and c, and its mass is m. If the values and possible errors in quantities a, b, c and m are as shown below, calculate the value of density and the possible error in this value.

$$a = 100\,\text{mm} \pm 1\%,\ b = 200\,\text{mm} \pm 1\%,\ c = 300\,\text{mm} \pm 1\%,\ m = 20\,\text{kg} \pm 0.5\%.$$

Solution

Value of $ab = 0.02\ \text{m}^2 \pm 2\%$ (possible error $= 1\% + 1\% = 2\%$)

Value of $(ab)c = 0.006\ \text{m}^3 \pm 3\%$ (possible error $= 2\% + 1\% = 3\%$)

Value of $\dfrac{m}{abc} = \dfrac{20}{0.006} = 3330\,\text{kg/m}^3 \pm 3.5\%$ (possible error $= 3\% + 0.5\% = 3.5\%$)

Or Possible error in Density : 1% +1% +1% + 0.5% = 3.5%

Main Types of Errors

- 1. Gross Errors
- 2. Systematic Error
  - – Instrumental Errors
  - – Environmental Errors
- 3. Observational Errors
- 4. Random Error

1. Gross Errors

- Refer to errors due to **human mistake** in reading instruments and recording and calculating measurement results.

- *Example*:

  a) Read the temperature as 31.5°C while the actual reading is 21.5 °C

  b) Read 25.8 °C but record as 28.5 °C

  c) 1 + 1 = 3 instead of 2

- **Prevention**: – Read and record carefully.

  – Take the average of several readings (at least 3 readings).

2. Systematic Errors

- **A. Instrumental Errors**

(i) Due to **inherent shortcomings** of the **instruments**

- Inherent due to their **mechanical** structure
- May be due to **improper calibration**, construction

. Due to **faulty** instrument.

- **To Overcome:**
  - Re-calibrate carefully
  - Apply correction factor after determining the instrumental errors
  - Proper maintenance, use and handling of instruments.

2. Systematic Errors

ii) Due to misuse of instrument

- Failure to **adjust** the zero of instrument (***calibrate***)
- Using **leads** of too high resistance (when measuring low R value)
- Measuring an ac signal which is beyond the instrument **bandwidth** response.

- To overcome:
  - Recalibrate system carefully
  - Use a right instrument for a particular application.

2. Systematic Errors

iii) Loading effect of instruments

- Due to the **internal resistance** of the device.
- Due to the **active filters** effect of the device.
- Improper ground or reference.

To overcome:

- Loading effect causes **inaccuracy** in measurement can be avoided by using appropriate instruments or using them intelligently.

2. Systematic Errors

B. Environmental Errors

- Effect due to temperature, pressure, humidity, dust, vibration or external magnetic or electrostatic fields.

- **To overcome:**

- Keep the **surrounding** conditions as constant as possible.
- Use equipment which **not affected** by these effects.
- Employ **techniques** which eliminate the effects of disturbances
- Apply computed **correction**

3. Observational Errors

- Observational error is the error due to **observation technique.**

- There are two **common types** of observational errors:

A) **Parallax error** – Error due to wrong observation angle/position.

B) **Reaction time error** –Error caused by the different instantaneous reaction of different people.

Example: Timer Watch

4. Random Error

- These are the errors due to **unknown** causes.

- These are un **avoidable** errors. Even though the devise been calibrated.

- To overcome this issue:
  - **More reading** need to be taken.
  - Using **statistical** means to obtain the best approximation to the true value.

Statistical Analysis

- Used to obtain the best approximation to the true value.

Statistical Analysis

Suppose an experiment were repeated many, say N, times to get, X_1, X_2, …., X_n

Arithmetic Mean

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{N} = \frac{\sum_{n-1}^{n=N} x_n}{N}$$

Deviation from the mean

$$d_i = x_i - \bar{x}$$

Average Deviation

$$D = \frac{|d_1| + |d_2| + \dots + |d_n|}{N} = \frac{\sum |d|}{n}$$

$$\sigma = \sqrt{\frac{d_1{}^2 + d_2{}^2 + ... + d_n{}^2}{N}}$$

Standard Deviations σ

$$\sigma = \sqrt{\frac{\sum_{n=1}^{N} (d_i)^2}{N}}$$

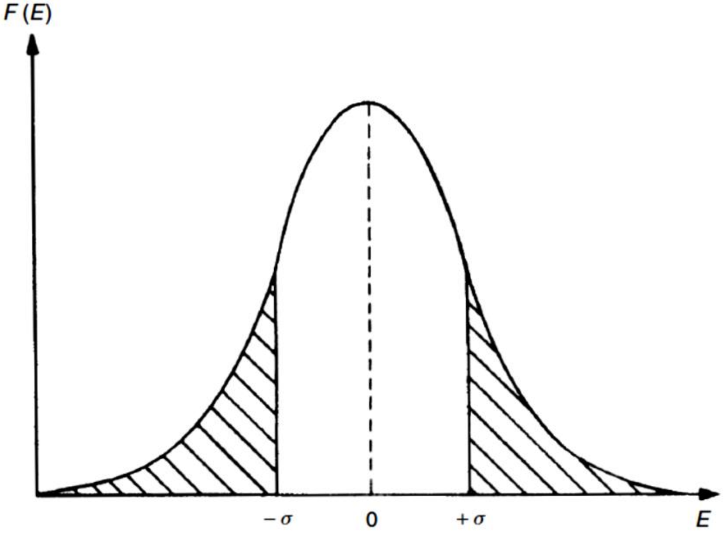
Practically, the number of readings are <u>finite</u>
So, the Standard Deviations σ Divided by N-1

$$\sigma = \sqrt{\frac{\sum_{n=1}^{N} (d_i)^2}{N-1}}$$

$$\sigma^2 = Variance$$

Probability of Errors

- The Gaussian (Normal) law of Errors form the basis for **probable error** analytical study.



- The most **probable true reading is the center or mean value.**

Probable Error (r)

- It is found that approx. 68% of of all the Data lies between the limits of $-\sigma$ and $+\sigma$ from the **mean.**
- So, for the case of large number of measurements the probable error (r) defined as

$$r = \pm 0.6745\sigma$$

Deviation boundaries	*% of data points within boundary*	*Probability of any particular data point being outside boundary*
$\pm\sigma$	68.0	32.0%
$\pm 2\sigma$	95.4	4.6%
$\pm 3\sigma$	99.7	0.3%

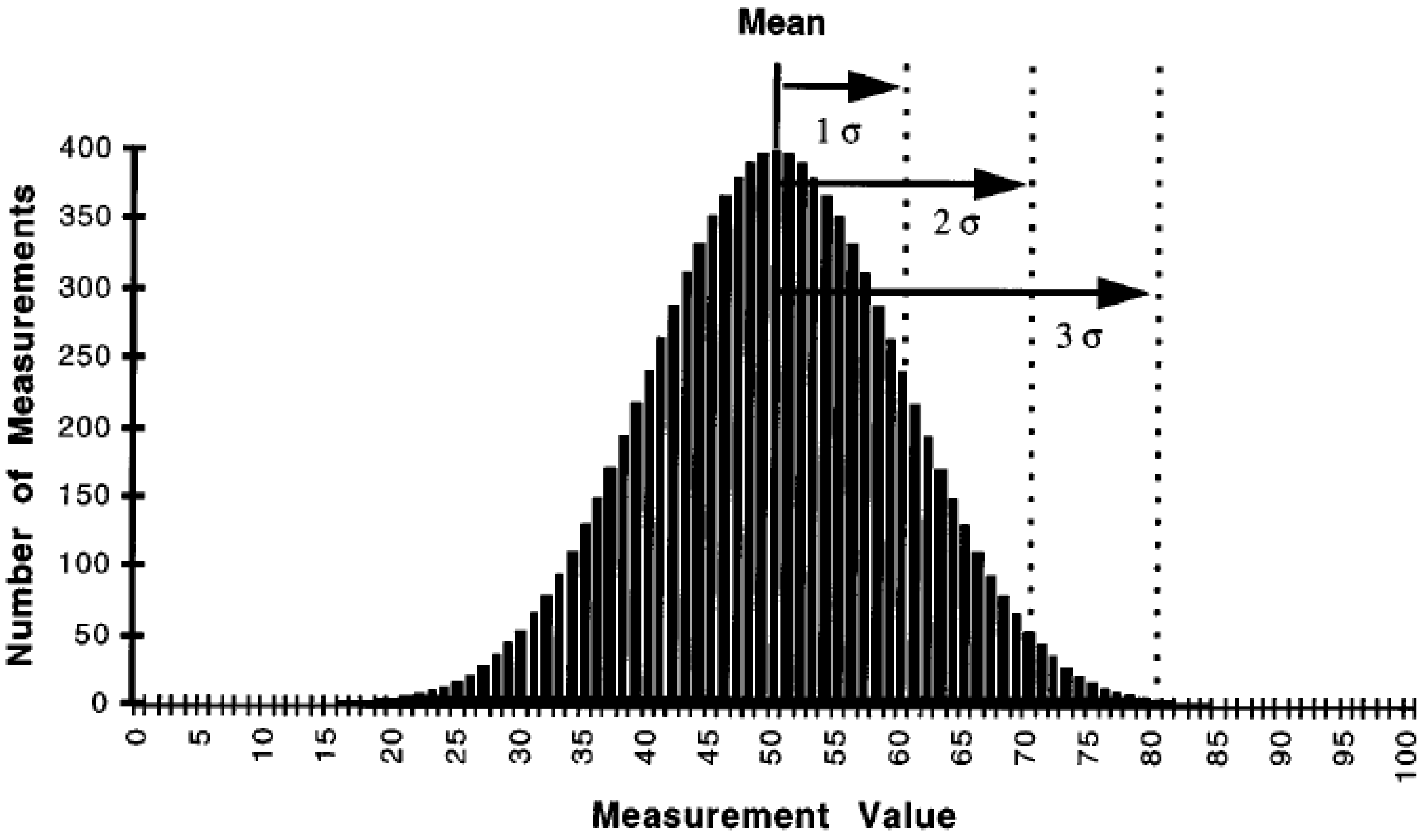


Table 1 shows a tabulation of **50 readings** of the voltage that were taken at small time intervals and recorded to the nearest **0.1 V**.
The nominal value of the measured voltage was **100.0 V.**

Voltage reading (volts)	Number of readings
99.7	1
99.8	4
99.9	12
100.0	19
100.1	10
100.2	3
100.3	1
	50

Table 1: Tabulation of Voltage Readings

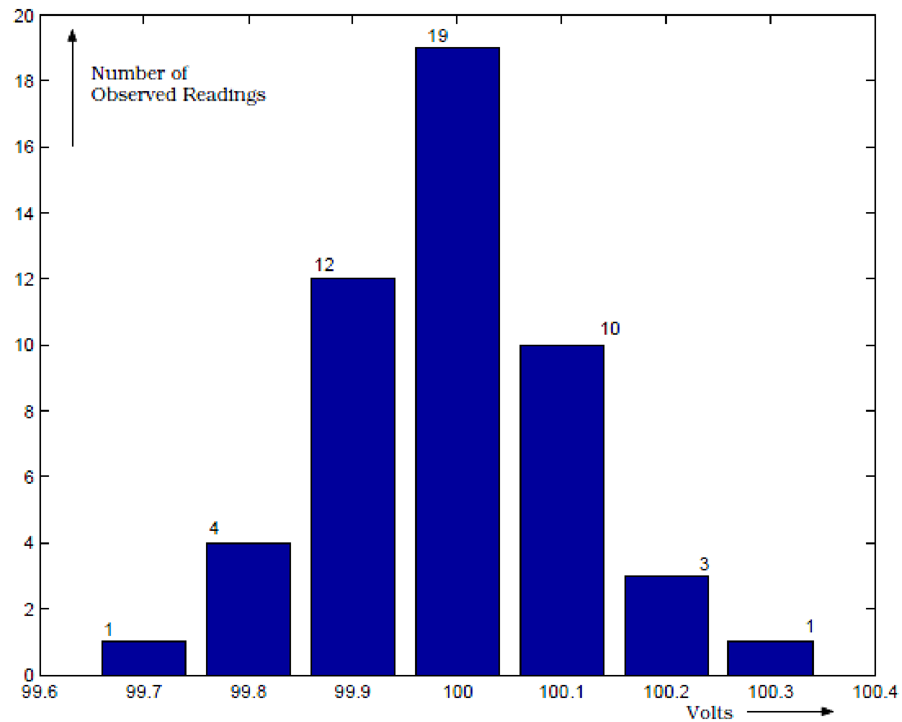


Figure 1 shows that the largest number of readings (19) occurs at the central value of 100.0 V,
while the other readings are placed more or less symmetrically on either side of the **central value**.

The error distribution curve of Figure 2 is based on the Normal law and shows a symmetrical distribution of errors.
This normal curve may be regarded as the limiting form of the histogram of Figure 1 in which the most probable value of the **true voltage is the mean value of 100.0 V**.

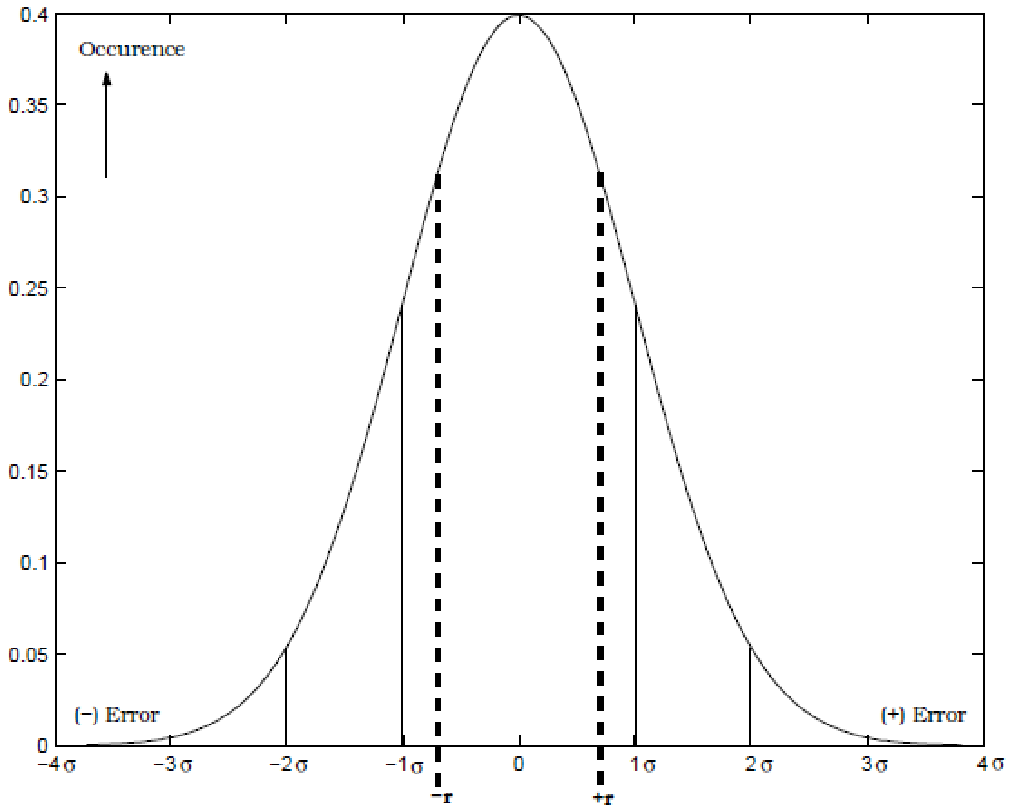


Figure 2: Curve for Normal Law. The highlighted portion(between bold dotted lines) indicates the region of probable error, where $r = \pm 0.6745\sigma$.

Example 11

Ten measurements of the resistance of a resistor gave 101.2 , 101.7 , 101.3 , 101.0 , 101.5 , 101.3 , 101.2 , 101.4 , 101.3 , and 101.1 .

Assume that only random errors are present. Calculate
1. the arithmetic mean,
2. the standard deviation of the readings,
3. the probable error.

Reading	Deviation	
x	d	d^2
101.2	-0.1	0.01
101.7	0.4	0.16
101.3	0.0	0.00
101.0	-0.3	0.09
101.5	0.2	0.04
101.3	0.0	0.00
101.2	-0.1	0.01
101.4	0.1	0.01
101.3	0.0	0.00
101.1	-0.2	0.04
$\Sigma x = 1{,}013.0$	$\Sigma \mid d \mid = 1.4$	$\Sigma d^2 = 0.36$

1. Arithmetic mean, $\overline{x} = \frac{\Sigma x}{n} = \frac{1013.0}{10} = 101.3\ \Omega$
2. Standard deviation, $\sigma = \sqrt{\frac{\Sigma d^2}{n-1}} = \sqrt{\frac{0.36}{9}} = 0.2\ \Omega$
3. Probable error $= 0.6745\sigma = 0.6745 \times 0.2 = 0.1349\ \Omega$